Decision Fusion Rules in Wireless Sensor Networks
Using Fading Channel Statistics

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Abstract — The problem of fusing decisions transmitted over fading and noisy channels in a wireless sensor network is revisited. In a previous work, starting with the assumption of perfect channel knowledge, an optimal likelihood ratio (LR) based fusion rule was developed along with three suboptimum statistics. In this paper, we present a new LR based fusion rule which requires only the knowledge of channel fading statistics instead of the actual channel coefficients. We show that the equal gain combiner (EGC) and the Chair-Varshney fusion rules are approximations of this new rule at low and high channel SNR values, respectively. This new fusion rule has better performance than Chair-Varshney and EGC fusion rules, outperforms the maximum ratio combiner (MRC) at most practical channel SNR values, and is only slightly worse than the optimal LR based fusion rule.

I. INTRODUCTION

Wireless sensor networks (WSN) have generated steadily growing interest recently, especially in the field of battlefield surveillance and environment monitoring. Much of the current research on WSN is focused on energy efficient routing protocols, distributed data compression and transmission, and collaborative signal processing[1]. The fact that the locally processed data is transmitted over a fading and noisy channel has not attracted much attention. Conventional means of employing channel coding and/or increased transmitter power may not be desirable due to the extremely limited system resources (both bandwidth and energy). In addition, there are always incentives to minimize transmission power in order to expand the life time of a WSN operating on a fixed energy budget. Signal processing and data fusion algorithms that can intelligently take into account the effects of fading channels may have the potential to achieve desirable performance with minimum resource consumption.

Consider a multiple-sensor decision fusion problem as illustrated in Fig. 1. A number of sensors collect and preprocess raw measurements, make local decisions, and transmit these decisions over fading and noisy channels to a fusion center. These local decisions are jointly processed at the fusion center where a final decision is made regarding the underlying inference problem, e.g., the presence or absence of a target. Numerous results on the distributed detection and decision fusion problem have been reported in the literature [2, 3, 4, 5, 6]. However, most studies assume that the local decisions can be transmitted to the fusion center without errors, i.e. they do not take into account the communication loss incurred due to channel fades and noise impairment. In [7, 8], an attempt was made to integrate the knowledge of the transmission channel into the decision fusion algorithm, resulting in channel aware fusion strategies. The optimal likelihood ratio (LR) based fusion rule requires the maximum amount of information, including complete channel knowledge and local sensors’ detection performance indices. From this, several sub-optimal alternatives were presented to relieve the stringent requirement of the optimal fusion rule [7, 8]. In particular, we showed that at low channel SNR, the optimal fusion rule reduces to a form reminiscent of a Maximum Ratio Combining (MRC) statistic for fading mitigation in wireless communications with channel diversity. At high SNR, the LR statistic reduces to a two stage processing structure that separates communication from the fusion algorithm. The fusion rule at the second stage is identical to the Chair-Varshney rule which assumes reliable access to the local decision output.

In this paper, we start from a more realistic channel model and derive its associated optimal fusion rule. Assuming Rayleigh flat fading channels, an LR based fusion rule is developed that requires only channel fading statistics instead of the actual channel coefficients. We show that EGC and Chair-Varshney rules are high and low SNR approximations.
of this new LR based fusion rule, respectively. Note that this EGC statistic was also proposed as a heuristic statistic in [7, 8] thus the new result provides a more rigorous theoretical justification. In addition, closed-form expressions for the global detection and false alarm probabilities for the Chair-Varshney rule are obtained. Analytical and simulation results confirm that, the new fusion rule outperforms both EGC and Chair-Varshney rules, and is only slightly degraded compared with the optimal LR fusion rule that assumes complete channel knowledge.

II. SYSTEM MODELING AND PREVIOUS FUSION RULES

A 3-Layer System Model

The 3-layer model for a distributed detection system in the presence of fading channels is illustrated in Fig. 1 and described in detail below.

- **Local sensor layer.**
  All $K$ local sensors collect observations generated under a specific hypothesis. We assume that the observations are independent of each other across sensors conditioned on any hypothesis. After receiving its observation, each sensor makes a hard (binary) decision: $u_k = 1$ is sent if $H_1$ is decided, and $u_k = -1$ is sent otherwise, where $k = 1, \cdots, K$. The detection performance of each local sensor node can be characterized by its corresponding probability of false alarm and detection, denoted by $P_{f_k}$ and $P_{d_k}$, respectively, for the $k$th sensor.

- **Fading channel layer.**
  Decisions at local sensors, denoted by $u_k$ for $k = 1, \cdots, K$, are transmitted over parallel channels that are assumed to undergo independent fading. In this paper, we assume flat Rayleigh fading channels between local sensors and the fusion center. We further assume phase coherent reception, thus the effect of fading channel is further simplified as a real scalar multiplication given that the transmitted signal is assumed to be binary. In the development of fusion rules, the amplitude of the fading channel is considered as a (possibly unknown) constant during the transmission of a single local decision. The channel noise is assumed as additive white Gaussian and uncorrelated from channel to channel. For simplicity, we assume that the noise variances are identical for different channels.

  Therefore, $y_k$, the output of the channel (or input to the fusion center) for the $k$th sensor is

  $$y_k = x_k + n_k$$
  $$= h_k u_k + n_k$$

  where $h_k$ is the attenuation of the fading channel and $n_k$ is a zero mean Gaussian random variable.

- **Fusion center.**
  Based on the received data $y_k$ for all $k$, the fusion center decides which hypothesis is more likely to be true. This is done by constructing a fusion statistic using the observations $y_k$ as well as some system parameters, if available, and comparing it with a threshold.

B Previous Fusion Rules

To facilitate our comparisons later, here we give a brief description of the fusion statistics proposed in [8].

B.1 Optimal LR based fusion rule

$$\Lambda = \sum_{k=1}^{K} \log \frac{P_{d_k} e^{-(y_k-h_k)^2/2\sigma^2} + (1 - P_{d_k}) e^{-(y_k+h_k)^2/2\sigma^2}}{P_{f_k} e^{-(y_k-h_k)^2/2\sigma^2} + (1 - P_{f_k}) e^{-(y_k+h_k)^2/2\sigma^2}}$$

(2)

where $\sigma^2$ is the variance of the additive white Gaussian noise for all channels. This fusion rule requires both local sensor performance indices and complete channel knowledge. We also proposed several suboptimum fusion rules that relieve the above requirements to some extent.

B.2 The Chair-Varshney fusion rule

$$\Lambda_1 = \sum_{\text{sign}(y_k)=1} \log \frac{P_{d_k}}{P_{f_k}} + \sum_{\text{sign}(y_k)=-1} \log \frac{1 - P_{d_k}}{1 - P_{f_k}}$$

(3)

We have shown that $\Lambda_1$ is mathematically equivalent to $\Lambda$ for the large SNR case, meaning that this statistic is a good approximation for the optimal $\Lambda$ for large channel SNR. $\Lambda_1$ does not require any knowledge regarding the channel statistics but does require $P_{d_k}$ and $P_{f_k}$ for all $k$.

B.3 Fusion rule using a maximum ratio combining statistic

We have shown that with small channel SNR and the assumption that $P_{d_k}$ and $P_{f_k}$ are the same for all $k$’s, $\Lambda$ in (2) reduces to a form analogous to a maximum ratio combiner [9]:

$$\Lambda_2 = \frac{1}{K} \sum_{k=1}^{K} h_k y_k$$

(4)

$\Lambda_2$ does not require the knowledge of $P_{d_k}$ and $P_{f_k}$. Knowledge of the channel gain is, however, required.

B.4 Fusion rule using an equal gain combining statistic

We proposed a third alternative in the form of an equal gain combiner (EGC) that requires minimum amount of information:

$$\Lambda_3 = \frac{1}{K} \sum_{k=1}^{K} y_k$$

(5)

Interestingly enough, this simple alternative outperforms both $\Lambda_1$ and $\Lambda_2$ for a wide range of SNR in terms of its detection performance.

III. CHANNEL STATISTICS BASED FUSION RULE

The optimal LR based fusion rule requires complete knowledge of the channel. However, for a WSN with very limited resources (energy and bandwidth), it is prohibitive to spend resources on estimating the channel every time a local sensor sends its decision to the fusion center. This motivates us to develop a new LR based fusion rule that requires only the knowledge of the statistics of the fading channel.

To obtain the likelihood ratio, we need to derive likelihood functions $f(y|H_1)$ and $f(y|H_0)$. First, consider $f(y|H_1)$. First of all, we have

$$p(u_k|H_1) = \begin{cases} P_{d_k} & u_k = 1 \\ 1 - P_{d_k} & u_k = -1 \end{cases}$$

(6)
We assume that the channel is Rayleigh fading with unit power (i.e. $E[h_k^2] = 1$) to facilitate SNR calculation later in this paper. Therefore, we have

$$f(h_k) = 2h_ke^{-h_k^2} \quad (7)$$

From (7) and the fact that $x_k = u_k h_k$, it is easy to get

$$f(x_k|u_k) = 2u_k x_k e^{-x_k^2/2} U(u_k x_k) \quad (8)$$

where $U(\cdot)$ is a step function defined as the following

$$U(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (9)$$

Because $n_k$ is Gaussian distributed with zero mean and variance $\sigma^2$, we also have

$$f(y_k|x_k) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(y_k-x_k)^2}{2\sigma^2}} \quad (10)$$

Thus, we have the following result:

$$f(y_k|u_k) = \int f(x_k|u_k) f(y_k|x_k, u_k) dx_k$$
$$= \int f(x_k|u_k) f(y_k|x_k) dx_k \quad (11)$$

where the identity $f(y_k|x_k, u_k) = f(y_k|x_k)$, which is obtained from the fact that $u_k, x_k, y_k$ form a Markov chain, has been used.

Next, the conditional pdf $f(y_k|u_k)$ is derived and stated as below:

### Lemma 1

The conditional pdf of $y_k$, the observation from sensor $k$, given the local decision $u_k$ is:

$$f(y_k|u_k) = \frac{2\sigma}{\sqrt{2\pi(1+2\sigma^2)}} e^{-\frac{y_k^2}{2\sigma^2}}$$

$$\times \left[ 1 + u_k \sqrt{2\pi ay_k e^{\frac{(ay_k)^2}{4}}} Q(-a y_k y_k) \right]$$

where $a = \frac{1}{\sqrt{1+2\sigma^2}}$, and $Q(\cdot)$ is the complementary distribution function of the standard Gaussian, i.e.,

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt.$$ 

### Proof

See Appendix A.

Once we have $f(y_k|u_k)$, it is easy to obtain $f(y_k|H_1)$, $f(y_k|H_0)$ and thus the likelihood ratio.

### Theorem 1

The log likelihood ratio with the knowledge of channel statistics and local detection performance indices is

$$\Lambda_4 = \log \left[ \frac{f(y_k|H_1)}{f(y_k|H_0)} \right]$$

$$= \sum_{k=1}^K \log \left\{ 1 + \frac{P_{d_k} - Q(ay_k)\sqrt{2\pi ay_k e^{\frac{(ay_k)^2}{4}}}}{1 + [P_{f_k} - Q(ay_k)\sqrt{2\pi ay_k e^{\frac{(ay_k)^2}{4}}}]} \right\} \quad (12)$$

### Proof

Under hypothesis $H_1$, we have

$$f(y_k|H_1) = \sum_{u_k} p(u_k|H_1) f(y_k|u_k) \quad (13)$$

$$= P_{d_k} f(y_k|u_k = 1) + (1 - P_{d_k}) f(y_k|u_k = -1)$$

$$= \frac{2\sigma}{\sqrt{2\pi(1+2\sigma^2)}} e^{-\frac{y_k^2}{2\sigma^2}} \times \left\{ 1 + [P_{d_k} - Q(ay_k)]\sqrt{2\pi ay_k e^{\frac{(ay_k)^2}{4}}} \right\}$$

where Lemma 1, Equation (6) and identity $Q(x) = 1 - Q(-x)$ have been used. Similarly, we obtain

$$f(y_k|H_0) = \frac{2\sigma}{\sqrt{2\pi(1+2\sigma^2)}} e^{-\frac{y_k^2}{2\sigma^2}} \times \left\{ 1 + [P_{f_k} - Q(ay_k)]\sqrt{2\pi ay_k e^{\frac{(ay_k)^2}{4}}} \right\} \quad (14)$$

Obviously, the likelihood ratio is

$$L_4 = \prod_{k=1}^K \frac{f(y_k|H_1)}{f(y_k|H_0)}$$

$$= \prod_{k=1}^K 1 + \frac{[P_{d_k} - Q(ay_k)]\sqrt{2\pi ay_k e^{\frac{(ay_k)^2}{4}}}}{1 + [P_{f_k} - Q(ay_k)]\sqrt{2\pi ay_k e^{\frac{(ay_k)^2}{4}}}} \quad (15)$$

where assumptions of the conditional independence of local decisions and the independence of different fading channels have been used.

Q.E.D.

Due to limited space, we will give two propositions without proof. The proof is straightforward when the property of $\lim_{\sigma^2 \to 0} \log(1+x) = x$ and the first order Taylor series expansion of $Q(x)$ are considered.

### Proposition 1

As the channel noise variance $\sigma^2 \to 0$, i.e., $SNR \to \infty$, $\Lambda_4$ defined in (12) reduces to $\Lambda_1$ defined in (3), i.e.,

$$\lim_{\sigma^2 \to 0} \Lambda_4 = \Lambda_1$$

Again, the new LR based statistic reduces to the decision statistic based on the Chair-Varshney rule at high channel SNR.

### Proposition 2

As $\sigma^2 \to \infty$, $\Lambda_4$ in (12) reduces to

$$\hat{A}_4 = \sum_{k=1}^K (P_{d_k} - P_{f_k}) y_k \quad (16)$$

Further, if the local sensors are identical, i.e., $P_{d_k}$ and $P_{f_k}$ are the same for all $k$’s, then $\Lambda_4$ further reduces to a form analogous to an equal gain combiner:

$$\hat{A}_3 = \frac{1}{K} \sum_{k=1}^K y_k \quad (17)$$

This is a very interesting observation. It implies that the EGC is a low-SNR approximation to the new decision fusion statistic requiring only the knowledge of channel statistics. Therefore, it gives a theoretical justification of the EGC statistic which was proposed as a heuristic alternative to the MRC statistic[8].

IV. Performance Analysis
The Distribution of the Chair-Varshney Statistic

Define $K_0 = |S_0|$, where $S_0 = \{k : y_k < 0\}$, i.e., $K_0$ is the cardinality of $S_0$; define $K_1 = |S_1|$, where $S_1 = \{k : y_k \geq 0\}$. Thus, $K_1 + K_0 = K$. With these definitions and assuming that $P_{d_k} = P_d$ and $P_{f_k} = P_f$ are the same for all the sensors, we observe that (3) becomes

$$
A_1 = \log \left( \frac{P_d}{P_f} \right) K_1 + \log \left( \frac{1 - P_d}{1 - P_f} \right) (K - K_1)
$$

$$
= \log \left( \frac{P_d (1 - P_f)}{P_f (1 - P_d)} \right) K_1 + \log \left( \frac{1 - P_d}{1 - P_f} \right) K
$$

which is an affine function of $K_1$. When $\frac{P_d (1 - P_f)}{P_f (1 - P_d)} > 1$ (or $P_d > P_f$), statistic $A_1$ is equivalent to $K_1$. When all the sensors are identical, all $y_k$s are independent and identically distributed (i.i.d.). Therefore, $K_1$ is a Binomial $(K,p)$ distribution, where $p$ is defined as

$$
p = \text{Prob}(y_k \geq 0)
$$

We derive the closed form solution for $p$ under two hypotheses, as stated in the following Lemma.

**Lemma 2** The probabilities of a non-negative observation $y_k$ under hypotheses $H_1$ and $H_0$ are

$$
p_1 = p(y_k \geq 0 | H_1) = \frac{1 + \frac{P_f - \frac{1}{2}}{\sqrt{1 + 2\sigma^2}}}{2}
$$

and

$$
p_0 = p(y_k \geq 0 | H_0) = \frac{1 + \frac{P_f - \frac{1}{2}}{\sqrt{1 + 2\sigma^2}}}{2}
$$

respectively.

**Proof:** See Appendix B.

System performance can be evaluated exactly by using the Binomial distribution, namely

$$
P_{d_0} = \sum_{i=K_r}^{K} \binom{K}{i} p_1^i (1 - p_1)^{K-i}
$$

where $K_r$ is the threshold. For the same $K_r$, the corresponding $P_{f_0}$ is

$$
P_{f_0} = \sum_{i=K_r}^{K} \binom{K}{i} p_0^i (1 - p_0)^{K-i}
$$

As we can see, there are a total of $K + 1$ pairs of $P_{d_0}$ and $P_{f_0}$ as $K_r$ takes values from 0 to $K$.

**B Asymptotic analysis**

If we assume that all the sensors are identical (thus $P_{d_k} = P_f$ and $P_{d_k} = P_d$ for all $k$), all the decision statistics we have discussed are sums of i.i.d. random variables which allows a direct application of the central limit theorem (CLT). Therefore, if the number of sensors is large, all these statistics can be approximated by Gaussian distributions. This makes the comparison and analysis much easier.

In order to use the CLT, we need the first and second order statistics (mean and variance). In the case of the LR based fusion rule with the knowledge of channel statistics, due to the complicated form of the $\Lambda_4$ (in Theorem 1) and the conditional pdf (in Lemma 1), it is nearly impossible to obtain closed form solutions of the conditional mean and variance. However, they are quite easy to calculate if we turn to numerical integration methods.

As far as the Chair-Varshney statistic, we have shown in Section A that $\Lambda_2$ is equivalent to $K_1$, which is Binomial distributed. Given $p_1$ and $p_0$, it is easy to show that

$$
\mu_{CV_1} = E[K_1 | H_1]
$$

$$
= K \left( \frac{1}{2} + \frac{P_d - \frac{1}{2}}{\sqrt{1 + 2\sigma^2}} \right)
$$

$$
\sigma^2_{CV_1} = \text{Var}[K_1 | H_1]
$$

$$
= K \left( \frac{1}{4} - \frac{(P_d - \frac{1}{2})^2}{1 + 2\sigma^2} \right)
$$

Similarly,

$$
\mu_{CV_0} = K \left( \frac{1}{2} + \frac{P_f - \frac{1}{2}}{\sqrt{1 + 2\sigma^2}} \right)
$$

$$
\sigma^2_{CV_0} = K \left( \frac{1}{4} - \frac{(P_f - \frac{1}{2})^2}{1 + 2\sigma^2} \right)
$$

Given the above statistics and Gaussian approximations, the probabilities of detection and false alarm can be easily calculated by using the $Q(\cdot)$ function.

Application of CLT allows more intuitive explanation and analysis on the performance of different fusion rules. Here we employ Stein’s lemma[10] to relate the detection performance to the relative entropy (also called Kullback Leibler distance) between the two distributions associated with the two hypotheses. According to Stein’s lemma, the relative entropy is the asymptotic error exponent, and larger relative entropy corresponds to better detection performance. For hypothesis testing between two Gaussian distributions, the relative entropy can be computed in a straightforward manner [8]. We calculate the relative entropy at different channel SNRs for different statistics and plot them in Fig. 2. As we can see, at very low channel SNR, $\Lambda_4$ is reduced to EGC; for high channel SNR, the Chair-Varshney statistic has the same performance as $\Lambda_4$. $\Lambda_4$ is optimal when only channel statistics and sensor performance indices are available. Note that when SNR is very low (less than -5 dB), MRC outperforms the new LR test. Again, that is because MRC has more prior information, namely the complete knowledge of the channel.

**C Simulation Results**

We have compared the performance of different fusion rules by analytical approximations using the CLT. However, CLT is only suitable to characterize small deviation and is not very accurate with a relatively small number of sensors. In this section, numerical simulations are provided to compare performances.

Figs. 3 and 4 give the receiver operating characteristic (ROC) curves of different fusion statistics at channel SNR of 0dB and 5dB, respectively. Each sensor’s false alarm rate is assumed to be $P_f = 0.05$ while the detection probability is $P_d = 0.5$. The total number of sensors is fixed at 8.
The optimal LR based fusion rule provides the uniformly most powerful detection performance, however it requires complete channel knowledge. On the other hand, its performance can be approached closely by the new channel statistics based LR test. Its performance is slightly worse than the optimal LR fusion rule for both SNR values and is better than the other schemes.

To better understand the performance difference as a function of channel SNR, Fig. 5 gives the probability of detection as a function of channel SNR for a constant system false alarm rate of $P_{fa} = 0.01$. The parameter setting is identical to the above example. From this figure, it is easy to see that at very low and very high SNR, performances of EGC and the Chair-Varshney fusion rules approach that of the new LR based fusion rule respectively, which is consistent with the results of Fig. 2 using the Kullback-Leibler divergence measure for the same set of parameters. The jumpy behavior of the Chair-Varshney approach is due to the finite alphabet property of the test statistics — with finite number of sensors (as is the case here), the values that the test statistic can take are from a finite set, which is obvious from the expression in (3).

V. Conclusions

Fusion of binary decisions transmitted over fading and noisy channels in the context of WSN is revisited in this paper. Along with the various statistics we have obtained previously, we present a new likelihood ratio based test which has optimal performance in the absence of complete channel knowledge (with the knowledge of only statistical characteristics of the wireless channel). The two-stage implementation using the Chair-Varshney fusion rule provides high SNR approximation to this new fusion rule, while the statistic in the form of a EGC gives a low SNR approximation.

Asymptotic analysis was carried out to demonstrate the performance advantage of the new fusion rule over other rules. Numerical simulation results are given for performance comparison and are consistent with the analysis.

A. Proof of Lemma 1

Substituting (8) and (10) into (11), and setting $u_k = 1$, we have

$$f(y_k|u_k = 1) = \frac{2}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} \int_0^\infty x e^{\frac{-(x-y)^2}{2\sigma^2}} dx$$

$$= \frac{2}{\sqrt{2\pi}\sigma} \int_0^\infty x e^{\frac{-(x-y)^2}{2\sigma^2}} \times$$

$$\int_0^\infty x e^{\frac{-(x-y)^2}{2\sigma^2}} dx$$

By plugging $t = x - \frac{y}{1+2\sigma^2}$ into the above equation, we have

$$f(y_k|u_k = 1) = \frac{2}{\sqrt{2\pi}\sigma} e^{-\frac{1}{1+2\sigma^2}} \times$$

$$\int_0^\infty (t + \frac{y}{1+2\sigma^2}) e^{\frac{-t^2}{2\sigma^2}} dt$$

$$= \frac{2\sigma}{\sqrt{2\pi}(1+2\sigma^2)} e^{-\frac{y^2}{2\sigma^2}} \times$$

$$\left[1 + \sqrt{2\pi ay e^{\frac{(ay)^2}{2}}} Q(-ay)\right]$$

where $a = \frac{1}{\sigma \sqrt{(1+2\sigma^2)}}$.

Similarly, we have

$$f(y_k|u_k = -1) = \frac{2\sigma}{\sqrt{2\pi}(1+2\sigma^2)} e^{-\frac{y^2}{2\sigma^2}} \times$$

$$\left[1 - \sqrt{2\pi ay e^{\frac{(ay)^2}{2}}} Q(ay)\right]$$

B. Proof of Lemma 2

$$p_1 = \int_0^\infty f(y_k|H_1)dy_k$$

$$= \int_0^\infty \frac{2\sigma}{\sqrt{2\pi}(1+2\sigma^2)} e^{-\frac{y^2}{2\sigma^2}} \times$$

$$\left\{1 + [P_d - Q(ay)] \sqrt{2\pi ay e^{\frac{(ay)^2}{2}}}\right\} dy$$

$$= \frac{2\sigma}{\sqrt{2\pi}(1+2\sigma^2)} (A + B + C)$$

(31)
where

\[ A = \int_0^\infty e^{-\frac{x^2}{2\sigma^2}} \, dx = \sqrt{\frac{2\pi}{\sigma^2}} \] (32)

\[ B = \int_0^\infty \sqrt{2\pi} P_0 \, a \, e^{-\frac{a^2 x^2}{2\sigma^2}} \, dx = \frac{\sqrt{2\pi} P_0 \sqrt{1 + 2\sigma^2}}{2\sigma} \] (33)

and

\[ C = -\int_0^\infty \sqrt{2\pi} Q(ay) \, a \, e^{-\frac{a^2 y^2}{1+2\sigma^2}} \, dy \] (34)

Substituting \( x = ay \) into the above equation, we have

\[ C = -\frac{\sqrt{2\pi}}{a} \int_0^\infty Q(x) \, x \, e^{-\frac{x^2}{2\sigma^2}} \, dx = \frac{\sqrt{2\pi}}{2a\sigma^2} \left[ e^{-\frac{a^2 x^2}{2\sigma^2}} Q(x) \right]_0^\infty - \int_0^\infty e^{-\frac{a^2 x^2}{2\sigma^2}} \, Q'(x) \, dx \]

\[ = \frac{\sqrt{2\pi}(1 - \sqrt{1 + 2\sigma^2})}{4\sigma} \] (35)

Therefore, we have

\[ A + B + C = \sqrt{2\pi} \frac{1 + 2\sigma^2 + (2P_d - 1)\sqrt{1 + 2\sigma^2}}{4\sigma} \] (36)

and

\[ p_1 = \frac{1}{2} + \frac{P_d - \frac{1}{2}}{\sqrt{1 + 2\sigma^2}} \] (37)

Following a similar procedure and replacing \( P_d \) with \( P_f \), it is easy to obtain

\[ p_0 = \frac{1}{2} + \frac{P_f - \frac{1}{2}}{\sqrt{1 + 2\sigma^2}} \] (38)

Fig. 4: ROC curves for various fusion rules for the Rayleigh fading channel with average channel SNR=5dB.

Fig. 5: Probability of detection as a function of channel SNR for Rayleigh fading channels with 8 sensors whose \( P_d = 0.5 \) and \( P_f = 0.05 \). The system false alarm rate is fixed at \( P_{f_0} = 0.01 \).

References


