Solution. 11.7:

Position: \[ x = 2t^3 - 12t^2 - 72t - 80 \text{ m} \] (1)

Velocity: \[ V = \frac{dx}{dt} = 6t^2 - 24t - 72 \text{ m/s} \] (2)

Acceleration: \[ a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = 12t - 24 \text{ m/s}^2 \] (3)

(a) To determine when velocity is zero:

From equation (2): \[ V = 6t^2 - 24t - 72 = 0 \]

Solving that equation: \[ t_1 = -2, \ t_2 = 6 \text{ s} \]

Since time cannot be negative, then:

\[ V = 0 \text{ when } t = 6 \text{ s} \]

For \( 0 < t < 6 \text{ s} \), \( V \) is negative, \( x \) is decreasing

For \( t > 6 \text{ s} \), \( V \) is positive, \( x \) is increasing

Minimum value of \( x \) occurs when \( t = 6 \text{ s} \).

\[ x_{\text{min}} = 2(6)^3 - 12(6)^2 - 72(6) - 80 = -512 \text{ m} \]

When \( t = 0 \), \( x_0 = -80 \text{ m} \)
(b) When \( x = 0 \): from equation (1):

\[ x = 2t^3 - 12t^2 - 72t - 80 = 0 \]

Solving that equation:

\[ t_1 = 8 \]
\[ t_2 = -2 \]
\[ t_3 = 10 \]

Neglecting negative times, \( x = 0 \) when \( t = 10 \) s

Using \( t = 10 \) in equation (2):

\[ v = 6t^2 - 24t - 72 = 6(10)^2 - 24(10) - 72 = 288 \text{ m/s} \]

\[ v = 288 \text{ when } t = 0 \]

Using \( t = 10 \) in equation (3):

\[ 0 = 12t - 8t = 12(10) - 24 = 96 \text{ m/s}^2 \]

\[ g = 96 \text{ when } x = 0 \]

To find the distance traveled at \( x = 0 \):

For \( 0 < t < 8 \) s:

\[ d_1 = |x_{10} - x_{0}| = |-512 - (-80)| = 432 \text{ m} \]

For \( 8 < t < 10 \) s:

\[ d_2 = |x_f - x_{10}| = |0 - (-512)| = 512 \text{ m} \]

Total distance traveled:

\[ d_1 + d_2 = 432 + 512 = 944 \text{ m} \]

\[ d_2 = 512 \text{ m} \]

\[ x_{10} = -512 \text{ m} \]
\[ x_0 = -80 \text{ m} \]
\[ x_f = 0 \]

\[ (t = 10 \text{ s}) \]
Solution 11.15.

Acceleration: \( a = 200x(1+kx^2) \text{ m/s}^2 \)  \( (1) \)

Since \( a \) is a function of \( x \), the equation to be used is:

\[
\frac{v^2}{2} = \int_{x_1}^{x_2} (200x + 200kx^2) \, dx
\]

Since \( V = V_1 = 2.5 \text{ m/s} \), \( x_1 = 0 \text{ m} \) and

\[
V = V_2 = 5 \text{ m/s} \quad x_2 = 0.15 \text{ m}
\]

\[
\frac{v^2}{2} \bigg|_{2.5}^{5} = \int_{0}^{0.15} (200x + 200kx^2) \, dx
\]

\[
5^2 - 2.5^2 = 100x^2 + 50kx^4 \bigg|_{0}^{0.15}
\]

\[
k = \frac{284.48 \text{ m}^2}{2}
\]
Solution 11.18.

Velocity: \[ v = 7.5 (1 - 0.04x)^{0.3} \text{ km/h} \] (1)

(a) To find the distance the jogger has run when \( t = 1 \) h,

since \( v = \frac{dx}{dt} \Rightarrow vdt = dx \) (2)

using equation (1) in equation (2):

\[ 7.5 (1 - 0.04x)^{0.3} \int_0^t dt = \int_0^x dx \]

\[ dt = \frac{dx}{7.5 (1 - 0.04x)^{0.3}} \]

\[ \int_0^t dt = \int_0^x \frac{0.133 (1 - 0.04x)^{0.3}}{0.7(1 - 0.04x^{0.7})} dx \] (3)

In the problem: \( t_0 = 0 \) at \( x_0 = 0 \) (given)

\( t = 1 \) at \( x = ? \)

Integrating equation (3):

\[ \int_0^1 \frac{0.133}{(0.7)(1 - 0.04x^{0.7})} dx \]

\[ x = 4.75 \left[ 1 - (1 - 0.04x^{0.7}) \right] \]

\[ x = 7.15 \text{ km} \]
(b) To find the acceleration when $t=0$,

using equation: $a = v \frac{dv}{dx}$

$$a = \frac{\left[7.5 (1-0.04x)^{0.3}\right] \left[7.5(0.3)(-0.04)(1-0.04x)^{0.7}\right]}{v} \frac{dv}{dx}$$

$$a = -0.675 (1-0.04x)^{0.3} (1-0.04x)^{-0.7}$$

$$a = -0.675 (1-0.04x)^{-0.4} \quad (4)$$

the question is to find acceleration when $t=0$. But since when $t=0$, $x=0$, by using $x=0$ in equation (4):

$$a = -0.675 \text{ km/h} = \frac{-0.675(1000)}{3600} = -5.2 \times 10^{-4} \text{ m/s}^2$$

$a = -5.21 \text{ m/s}$ when $t=0$.

(c) from equation (3)

$$\int_{t=0}^{t} dt = \int_{x=0}^{x} 0.132 (1-0.04x)^{-0.3} dx$$

$$t = 4.75 \left[ 1-(1-0.04x)^{0.7} \right]$$

for $x=6 \text{ km}$, $t = 4.75 \left[ 1-(1-0.04(6))^{0.7} \right] = 0.83 \text{ h} = 49.8 \text{ min}$
Velocity: \[ \mathbf{v} = \mathbf{v}_0 \left[ 1 - \sin \left( \frac{\pi t}{T} \right) \right] \]  

(a) To determine the position of the particle when \( t = 3T \),

\[ \frac{\mathbf{d}x}{\mathbf{d}t} = \mathbf{v}_0 \left[ 1 - \sin \left( \frac{\pi t}{T} \right) \right] \Rightarrow \frac{\mathbf{d}x}{\mathbf{d}t} = \mathbf{v}_0 \left[ 1 - \sin \left( \frac{\pi t}{T} \right) \right] \]

\[ \int_{0}^{x} \mathbf{d}x = \int_{0}^{t} \mathbf{v}_0 \left[ 1 - \sin \left( \frac{\pi t}{T} \right) \right] \mathbf{d}t \]

\[ x = x_0 + \mathbf{v}_0 \left[ t + \frac{T}{\pi} \cos \left( \frac{\pi t}{T} \right) \right] \]

\[ x = v_0 t + \frac{v_0}{\pi} \cos \left( \frac{3\pi}{T} \right) - \frac{v_0 T}{\pi} \]

when \( t = 3T \) \[ x = 3Tv_0 + \frac{v_0 T}{\pi} \cos \left( \frac{3\pi T}{T} \right) - \frac{v_0 T}{\pi} \]

\[ x = 2.36v_0 T \]

To determine the acceleration of particle when \( t = 3T \),

\[ \mathbf{a} = \frac{\mathbf{d}\mathbf{v}}{\mathbf{d}t} = \mathbf{v}_0 \left[ 1 - \sin \left( \frac{\pi t}{T} \right) \right] \text{ for } t = 3T \]

\[ a = v_0 \frac{\pi}{T} \cos 3\pi T \]

\[ a = \frac{v_0 \pi}{T} \]
(b) Using equation (2) with $t = T$

$$X_T = V_0 T + \frac{V_0 T}{\pi} \cos \frac{\pi T}{T} = V_0 T \left(1 - \frac{2}{\pi}\right)$$

**Average velocity:**

$$V_{ave} = \frac{\Delta x}{\Delta t} = \frac{X_T - X_0}{T - 0} = \left(1 - \frac{2}{\pi}\right) V_0 = 0.363 V_0$$
Solution 11.33:

\[ v_0 = v_A = 0 \quad x_0 = x_A = 0 \]

Since the acceleration is constant,

\[ a = \frac{dv}{dt} \Rightarrow v = v_0 + at = at \quad (1) \]

\[ v = \frac{dx}{dt} \Rightarrow \int dt = \int \frac{dx}{v} \Rightarrow x = x_0 + \frac{1}{2} at^2 = \frac{1}{2} at^2 \quad (2) \]

At point B: \( x_B = 900 \text{ m} \) and \( t = 30 \text{ s} \)

(a) The acceleration:

From equation (2): \( x = \frac{1}{2} at^2 \Rightarrow 900 = \frac{1}{2} a (30)^2 \)

\[ a_B = 2 \text{ m/s}^2 \]

(b) The velocity:

From equation (1) \( v = at \) since \( a = 2 \) and \( t = 30 \)

\[ v_B = (2)(30) = 60 \text{ m/s} \]
Solution: M.L.H.

(a) To find the acceleration of A,

\[ v_A = v_{Ao} + a_A t \]

\[ x_A = x_{Ao} + v_{Ao} t + \frac{1}{2} a_A t^2 \]

But since \( v_{Ao} = 0 \) and \( x_{Ao} = 0 \) therefore:

\[ v_A = a_A t \quad \text{and} \quad x_A = \frac{1}{2} a_A t^2 \]

when cars pass each other (let us say at time \( t = t_1 \)):

\[ x_A = 0 \] \( \Rightarrow \) \[ 0 = \frac{1}{2} a_A t_1^2 \] \( \Rightarrow \) \[ t_1^2 = \frac{180}{a_A} \] \( \quad \text{(1)} \)

\[ v_A = a_A t_1 \] \( \quad \text{(2)} \)

for \( t < 0 < s \), \( v_B = v_{Bo} = -96 \text{ km/hr} = -26.667 \text{ m/s} \)

for \( t > s \), \( v_B = v_{Bo} + a_B (t-s) = -26.667 + \frac{1}{6} a_A (t-s) \) \( \quad \text{(3)} \)

when vehicles pass: \( v_B = -v_B \)

from equation (2) and (3): \( a_A t_1 = 2 \times 26.667 - \frac{1}{6} a_A (t_1-s) \)

\[ a_A t_1 - \frac{1}{6} a_A t_1 = 2 \times 26.667 + \frac{5}{6} a_A \]

\[ t_1 = \frac{160}{a_A} + 5 \] \( \quad \text{(5)} \)

using equation (1) in equation (5)

\[ t_1 = \sqrt{\frac{160}{a_A}} + s \] \( \quad \text{(6)} \)
Solving equation (5) (this equation is a quadratic equation and has 2 roots)

\[ a_1 = 2.852 \text{ m/s}^2 \quad \text{and} \quad 3.590 \text{ m/s}^2 \]

To decide which value of \( a_1 \) is right, let us look for corresponding values of \( t_1 \)

For \( a_1 = 2.852 \), \( t_1 = \frac{190}{2 \times 2.852} = 0.794 \text{ s} \)

For \( a_1 = 3.590 \), \( t_1 = \sqrt{\frac{190}{3.590}} = 7.08 \text{ s} \)

Reject 0.794 since it is less than 5 s, thus \( a_1 = 3.590 \text{ m/s}^2 \)

b) Time of passing \( t = t_1 = 7.08 \text{ s} \)

c) To find distance \( d \):

\[ 0 < t < 5 \text{ s} \quad : \quad x_B = x_0 - v_{0y}t = d - 2b \cdot 667t \text{ m} \]

At \( t = 5 \text{ s} \) : \( x_B = d - 2b \cdot 667 (5) = d - 133.33 \text{ m} \)

For \( t > 5 \text{ s} \) : \( x_B = (d - 133.33) + v_{0y} (t-5) + \frac{1}{2} a_y (t-5)^2 \)

\[ x_B = d - 133.33 - 2b \cdot 667(t-5) + \frac{1}{2} \left( \frac{3.590}{6} \right) (t-5)^2 \]

When \( t = t_1 = 7.08 \) \( x_A = x_B = 90 \)

\[ 90 = d - 133.33 - 2b \cdot 667(7.08-5) + \frac{1}{2} \left( \frac{3.590}{6} \right) (7.08-5)^2 \]

\[ d = 278 \text{ m} \]
Solution. 11.47.

Let \( x \) be the positive downward for all blocks and for point 0;

from the figure:

constraint of cable supporting \( A \):
\[
x_A + (x_A - y_B) = \text{constant}
\]
\[
2x_A - y_B = \text{constant}
\]
\[
2v_A - v_B = 0
\]

Since \( v_A = 2 \text{ ft/sec} \) \( \Rightarrow v_B = 2v_A = 4 \text{ ft/sec} \)

(a) constraint of cable supporting \( B \):
\[
2x_B + x_C = \text{constant}
\]
\[
2v_B + v_C = 0
\]
\[
v_C = -2v_B = -8 \text{ ft/sec}
\]

(b) \( v_{B/A} = v_B - v_A = 4 - 2 = 2 \text{ feet/sec} \)

(c) from the figure:
\[
x_C + x_D = \text{constant}
\]
\[
v_C + v_D = 0 \Rightarrow v_D = -v_C = 8 \text{ ft/sec}
\]
\[
v_{D/A} = v_D - v_A = 8 - 2 = 6 \text{ feet/sec}.
\]
Solution 11.54:

Let $x$ be position relative to the right support increasing to the left.

Constraint of entire cable:

$$2x_A + x_B + (x_B - x_A) = \text{constant}$$

$$x_A + 2x_B = \text{constant}$$

$$v_A + 2v_B = 0 \Rightarrow v_A = -2v_B$$  \hspace{1cm} (1)

Constraint of point $C$ of cable:

$$2x_A + x_C = \text{constant}$$

$$2v_A + v_C = 0 \Rightarrow v_C = -2v_A$$  \hspace{1cm} (2)

(a) Velocity of collar $A$:

From equation (1):

$$v_A = -2v_B = (-2)(300 \text{ mm/sec}) = -600 \text{ mm/sec}$$

(b) Velocity of point $C$ of the cable:

From equation (2):

$$v_C = -2v_A = (-2)(-600 \text{ mm/sec}) = 1200 \text{ mm/sec}$$

(c) Velocity of point $C$ relative to collar $B$:

$$v_{C/B} = v_C - v_B = 1200 - 300 = 900 \text{ mm/sec}$$
Solution:

Given:

\[ x_0 = 0, \quad t_0 = 0 \]

\[ v_b = -8 \text{ ft/s} \]

For constant acceleration cases:

\[ v = v_0 + at \]

For \( 0 < t < b \), \( v = v_0 - bt \) \hspace{1cm} (1)

For \( t = b \), \[ v = -8 = v_0 - 4(b) \Rightarrow v_0 = +16 \text{ ft/s} \]

Using \( v_0 = 16 \text{ m/s} \) in equation (1):

For \( 0 < t < b \), \[ v = 16 - 8t \text{ ft/s} \] \hspace{1cm} (2)

For \( b < t < 14 \), \[ v = -8 \text{ ft/s} \] \hspace{1cm} (3)

For \( 10 < t < 14 \), \[ v = v_{10} + a(t-10) \]

\[ = -8 + 4(t-10) \] \hspace{1cm} (4)

For \( t = 14 \), \[ v_{14} = -8 + 4(4) = 8 \text{ ft/s} \] \hspace{1cm} (5)

From equations (2)-(5): \( v-t \) curve:

\[ \quad \]
for \( x-t \) curve:

\[ x = x_0 + v_0 t + \frac{1}{2} a t^2 \quad \Rightarrow x_0 = 0 \]  
\[ \text{for } 0 < t < b : x = x_0 + v_0 t + \frac{1}{2} a t^2 = 16t - 2t^2 \]  
\[ \text{for } t = b : x_b = 16 (b) - 2 (b)^2 = 24 \text{ ft} \]  
\[ \text{for } b < t < 10 : x = x_b + v_b (t-b) = 24 - 8 (t-b) \]  
\[ \text{for } t = 10 : x_{10} = 24 - 8 (10 - b) = -8 \text{ ft} \]  
\[ \text{for } 10 < t < 14 : x = x_{10} + v_{10} (t-10) + \frac{1}{2} a (t-10)^2 \]
\[ = -8 - 8 (t-10) + \frac{1}{2} 4 (t-10)^2 \]  
\[ \text{for } t = 14 : x_{14} = -8 - 8(4) + 2 (4)^2 = -8 \text{ ft} \]  

From equations (6)-(12) \( x-t \) curve:

\[ \text{at } t = 14, \quad v_{14} = 8 \text{ ft/s} \quad (\text{equation } 5) \]
\[ x_{14} = -8 \text{ ft} \quad (\text{equation } 12) \]

Total distance traveled:
\[ \text{for } 0 < t < 4 : d_1 = |32 - 0| = 32 \text{ ft} \]
\[ \text{for } 4 < t < 12 : d_2 = |16 - 32| = 16 \text{ ft} \]
\[ \text{for } 12 < t < 14 : d_3 = |14 - (-14)| = 8 \text{ ft} \]

Total distance = \( d_1 + d_2 + d_3 = 56 \text{ ft} \)
Solution 11.62.

(a) Part (a) of this problem is same as part (a) of the problem 11.61.

(b) Time for $|x| > 16$ ft, using $x$-$t$ graph we have in problem 11.61

![Graph showing motion](image)

we are looking for $|t_2 - t_1| = ?$

from equation 1b in solution 11.61

for $0 < t < 6 s \Rightarrow x = 16t - 2t^2$

we know that for $t = t_1$, $x = 16$. Thus:

$16 = 16t_1 - 2t_1^2 \Rightarrow t_1^2 - 8t_1 + 8 = 0$

$t_1 = 6.828$ and $t_1 = 1.172$

since $6.828 > 6$ then $t_1 = 1.172$

from equation (b) in solution 11.61:

for $6 < t < 10 s \Rightarrow x = 24 - 8(t - b)$

Required time interval:

for $t = t_2$, $x = 16 \Rightarrow 16 = 24 - 8(t_2 - b) \Rightarrow t_2 = 7 s \Rightarrow b - t_1 = 7 - 1.172 = 5.83 s$
Solution 11.65.

Let us sketch the v-t and x-t curve by using information given in the problem

when \( t = t_0 = 0 \), \( V_0 = -180 \text{ ft/s} \)
\( x_0 = 1500 \text{ ft} \)

Since:
\[
x = x_0 + V_0 t + \frac{1}{2} a t^2
\]

for \( 0 < t < t_1 \):
\[
x = 1500 - 180 t + \frac{1}{2} a t^2
\]

for \( 0 < t < t_1 \):
\[
a_0 = \frac{\Delta v}{\Delta t} = \frac{-441 - 180}{t_1} = \frac{135}{t_1}
\]

for \( t > t_1 \):
\[
x = 1800 = 1500 - 180 t_1 + \frac{1}{2} a_0 t_1^2
\]

\[
-100 = -180 t_1 + \frac{1}{2} \frac{135}{t_1} t_1^2
\]

\[
t_1 = 0.993 \text{ sec}
\]

for \( t_1 < t < t_2 \):
\[
x = x_1 + V_1 t + \frac{1}{2} a_{1-2}
\]

\[
\sin a_{1-2} = 0 \quad (V_{1-2} \text{ is constant})
\]

for \( t_1 < t < t_2 \):
\[
x = 1800 - 44 (t_2 - t_1)
\]

for \( t = t_2 \):
\[
x_2 = 100 = 1800 - 44 (t_2 - t_1)
\]

\[
-1700 = -44 (t_2 - t_1) \quad \Rightarrow \quad t_2 = 32.53 \text{ s}
\]

for \( t_2 < t < t_3 \):
\[
x = x_2 + V_2 (t - t_2) + \frac{1}{2} a_2 (t_3 - t_2)^2
\]

for \( t = t_3 \):
\[
a_{2-3} = \frac{V_3 - V_2}{t_3 - t_2} = \frac{44}{t_3 - t_2}
\]
for \( t = t_3 \) : \( x = 0 = 100 - 44 (t_3 - t_2) + \frac{1}{2} \frac{44}{(t_3 - t_2)} (t_3 - t_2)^2 \)

\[ 0 = 100 - 22(t_3 - t_2) \]

\[ t_3 = \frac{100}{22} + t_2 = 44.07 \text{ sec} \]

(a) Total time = \( t_3 = 44.07 \text{ sec} \).

(b) Initial acceleration = \( a_0 = \frac{\Delta V}{\Delta t} = \frac{V_1 - V_0}{t_1} = \frac{(-44) - (-180)}{0.893} \]

\[ = 152.3 \text{ ft/s}^2 \]
Solution 11.82:

Approximate the curve by a series of rectangles of height \( a_i \), each with its centroid at \( t = t_i \). When equal widths of \( \Delta t = 2s \) are used, the values of \( b_i \) and \( a_i \) are those shown in the first two columns of the table below.

<table>
<thead>
<tr>
<th>( t_i )</th>
<th>( a_i )</th>
<th>( b_i - t_i )</th>
<th>( a(20-t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.58</td>
<td>19</td>
<td>124.0</td>
</tr>
<tr>
<td>2</td>
<td>19.14</td>
<td>17</td>
<td>238.0</td>
</tr>
<tr>
<td>3</td>
<td>7.74</td>
<td>13</td>
<td>100.6</td>
</tr>
<tr>
<td>4</td>
<td>6.18</td>
<td>11</td>
<td>68.0</td>
</tr>
<tr>
<td>5</td>
<td>5.13</td>
<td>9</td>
<td>46.2</td>
</tr>
<tr>
<td>6</td>
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<td>7</td>
<td>29.8</td>
</tr>
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<td>7</td>
<td>3.69</td>
<td>5</td>
<td>18.5</td>
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<td>1</td>
<td>3.0</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( A \) at \( t = 8s \), \( v_8 = v_0 + \int_{0}^{8} a dt = 0 + \sum a_i \Delta t = (\sum a_i) \Delta t \)

Since \( t = 8 \), only first four values in the second column are summed.

\( \sum a_i = 11.58 + 13.14 + 10.14 + 7.74 = 42.67 \text{ ft/s}^2 \)

\( v_8 = (\sum a_i) \Delta t = (42.67) \times (2) = 85.34 \text{ ft/s} \)

\( B \) at \( t = 20s \):

\( x_{20} = v_{20}t + \int_{0}^{20} a(20-t) \, dt = 0 + \sum a_i (20-t) \Delta t \)

\( = (990.4) \times (2) = 1980 \text{ ft} \)