Extensions to the Preferred Ordering Theorem for Sequential Radar Updating

Donald M. Leskiw (student) and Hong Wang (professor)
Syracuse University
dmleskiw @syr.edu and hongwang @syr.edu

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Outline

- Preferred Ordering
  - Extensions to the Preferred Ordering Theorem
- Motivation
  - Radar tracking system
  - Examples
- The Basic Tracking Equations (the BLUE)
  - Commutativity
- Nonlinear Case
  - Batch versus Sequential
  - Non-commutativity
- References

The Preferred Ordering Theorem

- Enables the EKF to be used directly
  - Allows defective detections to be used (*R* only or *A* only; also *E*)
- Most convert to Cartesian and update there
  - Not possible with defective detections, or Doppler
- For EKF, most prefer *R*, *E*, *A*
  - Actually, the preferred ordering is *A*, *E*, *R*
- Efficacy wanes, however, as track converges

$$\hat{\mathbf{x}}_{AR} = \hat{\mathbf{x}}_{A} + \mathbf{K}_{AR}\left[R - r(\hat{\mathbf{x}}_{A})\right] \text{ and } \mathbf{P}_{AR} = \left[\mathbf{I} - \mathbf{K}_{AR}\mathbf{h}_{AR}^{T}\right]\mathbf{P}_{A}$$

$$\mathbf{K}_{AR} = \mathbf{P}_{A}(-)\mathbf{h}_{AR}^{T} \Big/ \left[\mathbf{h}_{AR}\mathbf{P}_{A}\mathbf{h}_{AR}^{T} + \sigma_{R}^{2}\right]$$

Extensions: Current Research

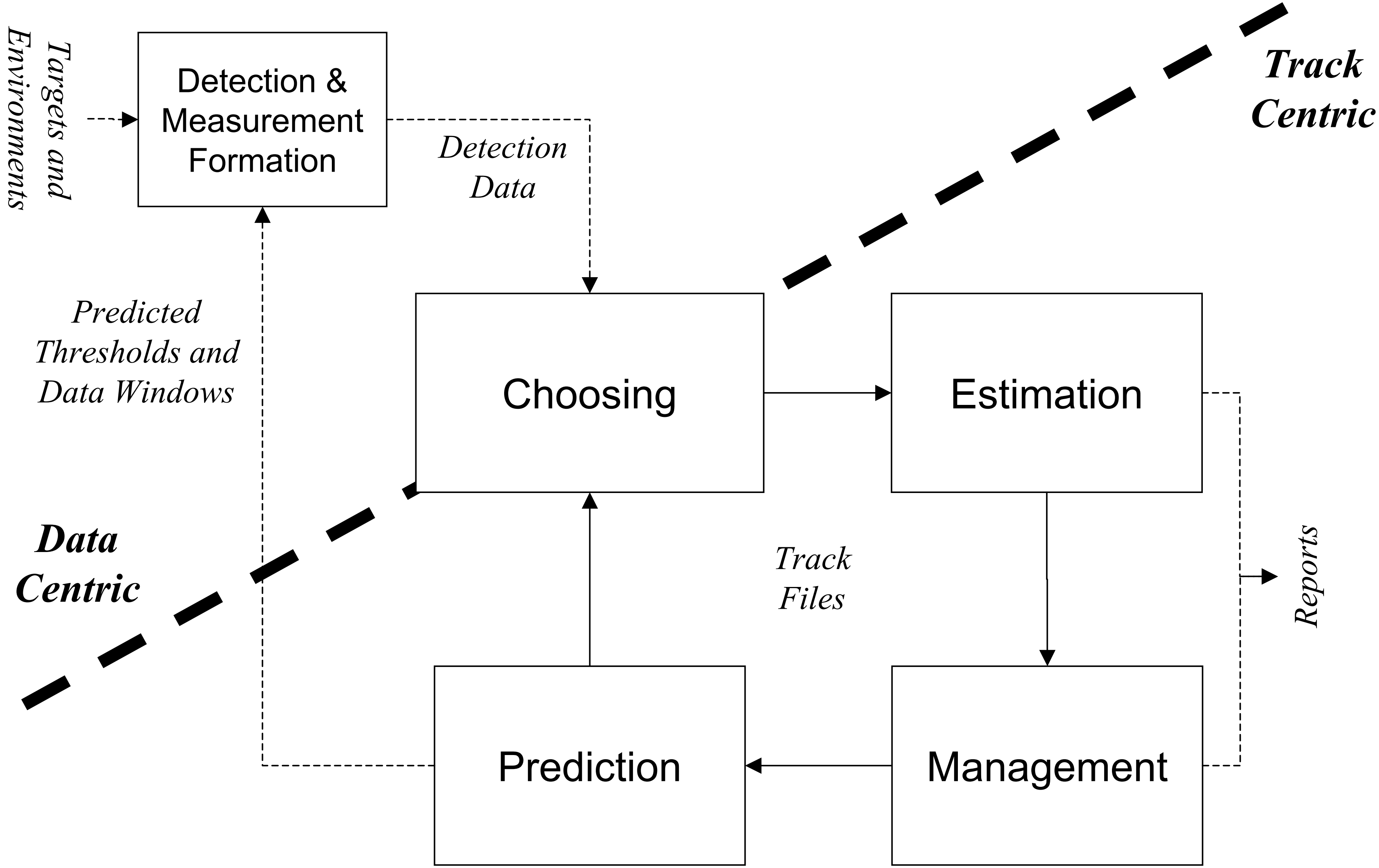
- Extending Theorem to Retain Efficacy as Track Converges
  - Performance is now independent of track accuracy (or measurement accuracy)
- Extending Theorem to Include Preferred Ordering of
  - Range Rate and
  - Phase-Derive Range and Rate Measurements
- Results in a forthcoming paper (Winter 2007)
  - Today, the motivation and context

Real TWS Radar Systems …

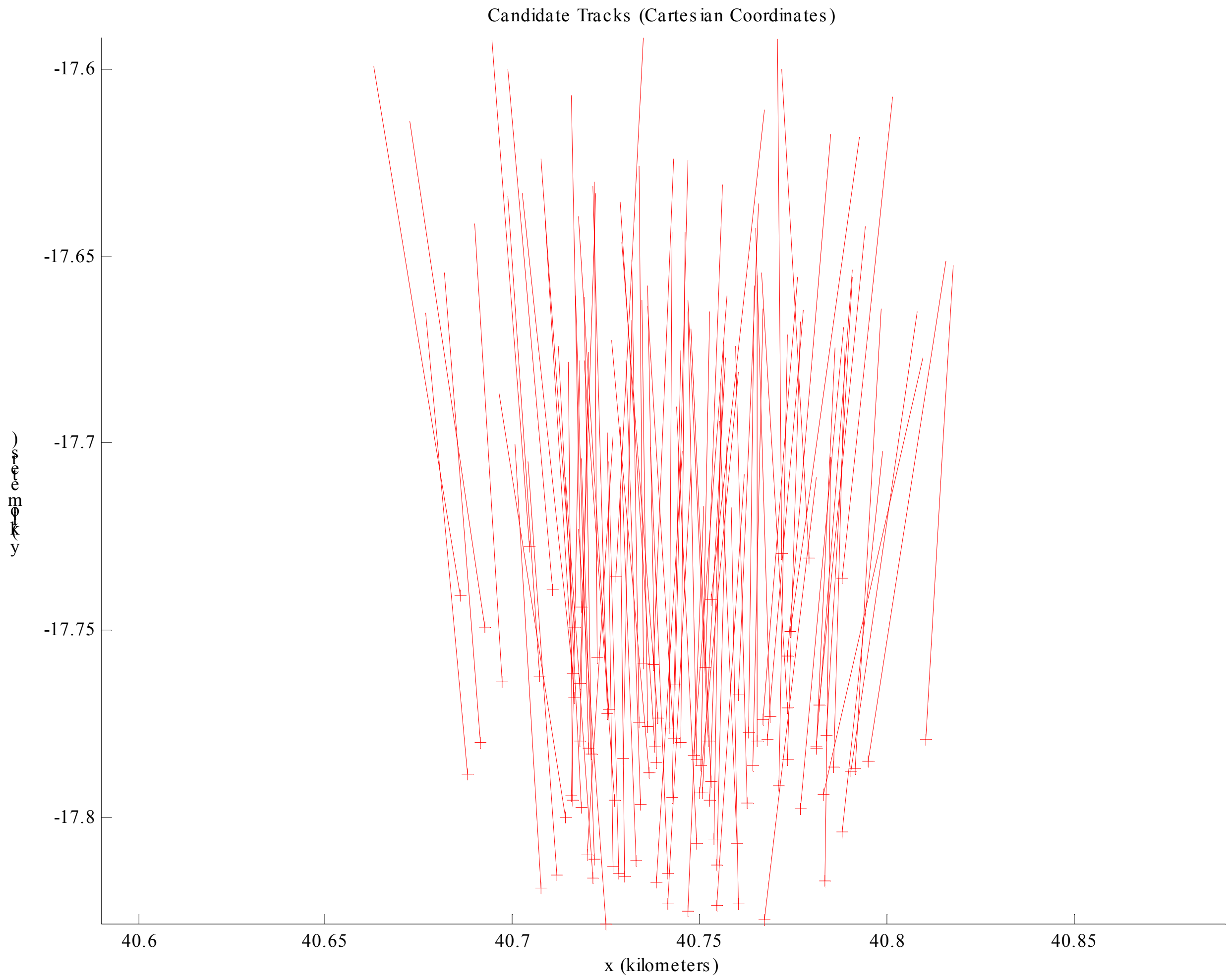
- A compromise solution to surveillance and tracking
  - Two systems in one
  - Slow, mechanical scan in azimuth; electronic scan in elevation
- Examples:

AN/TPS-59（V）3B弹道导弹雷达　　意大利空军升级FPS-117型远程雷达性能

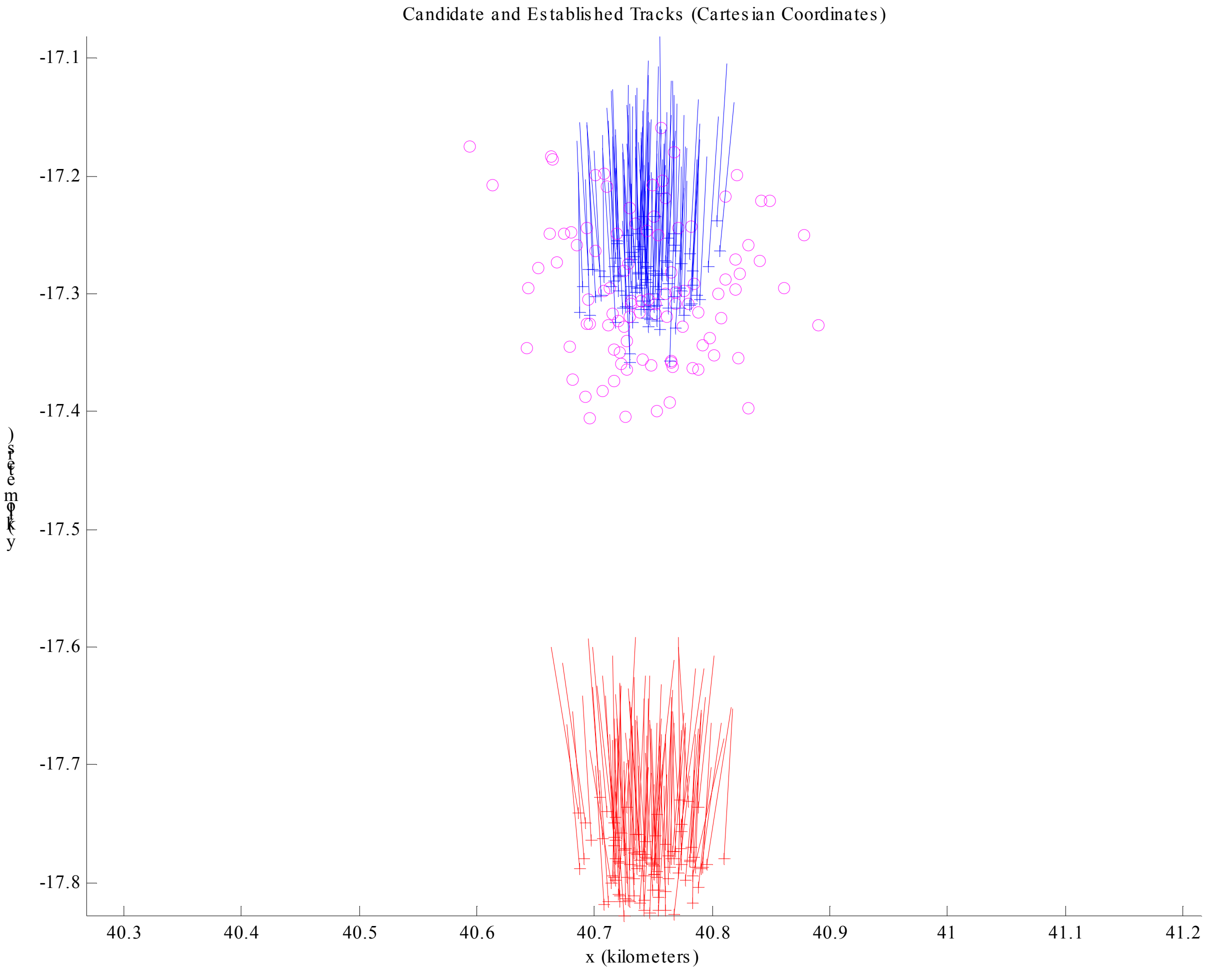
Radar Tracking – The Big Loop



Initialization (1 of 2)

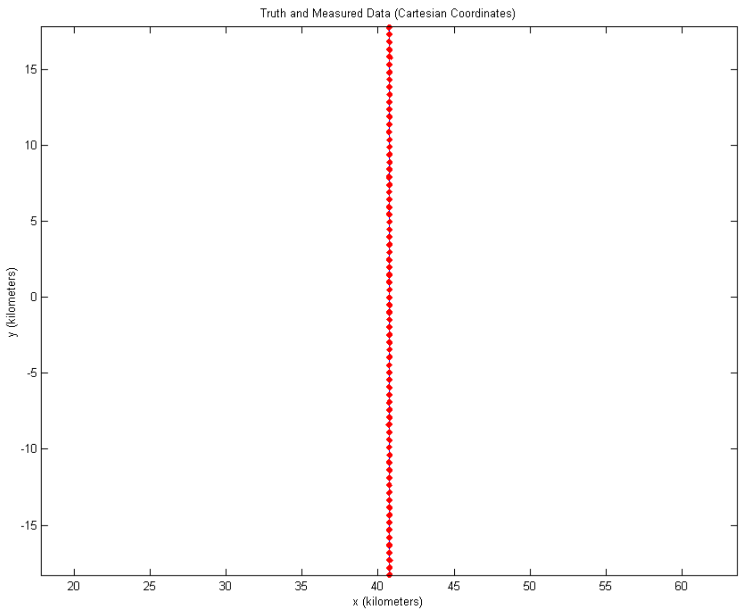


Initialization (2 of 2)

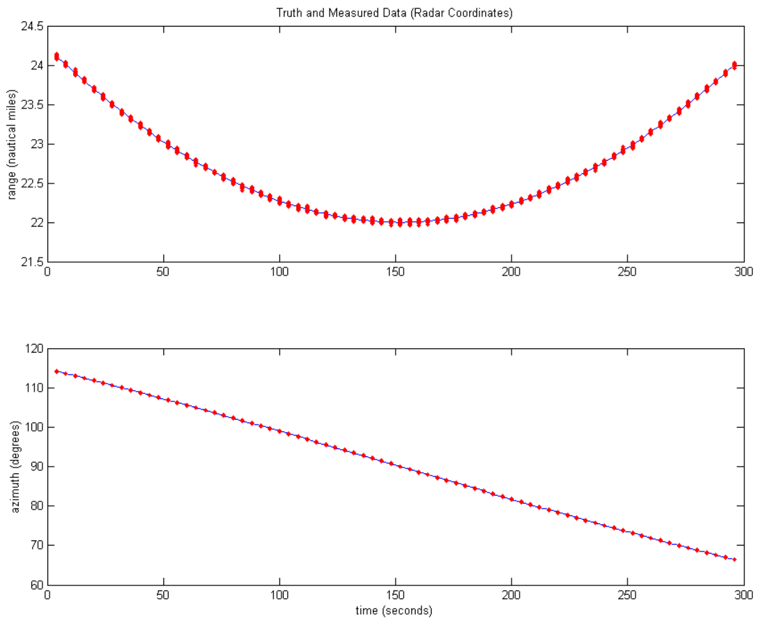


2 DOF CV Case: Effect of Nonlinearity

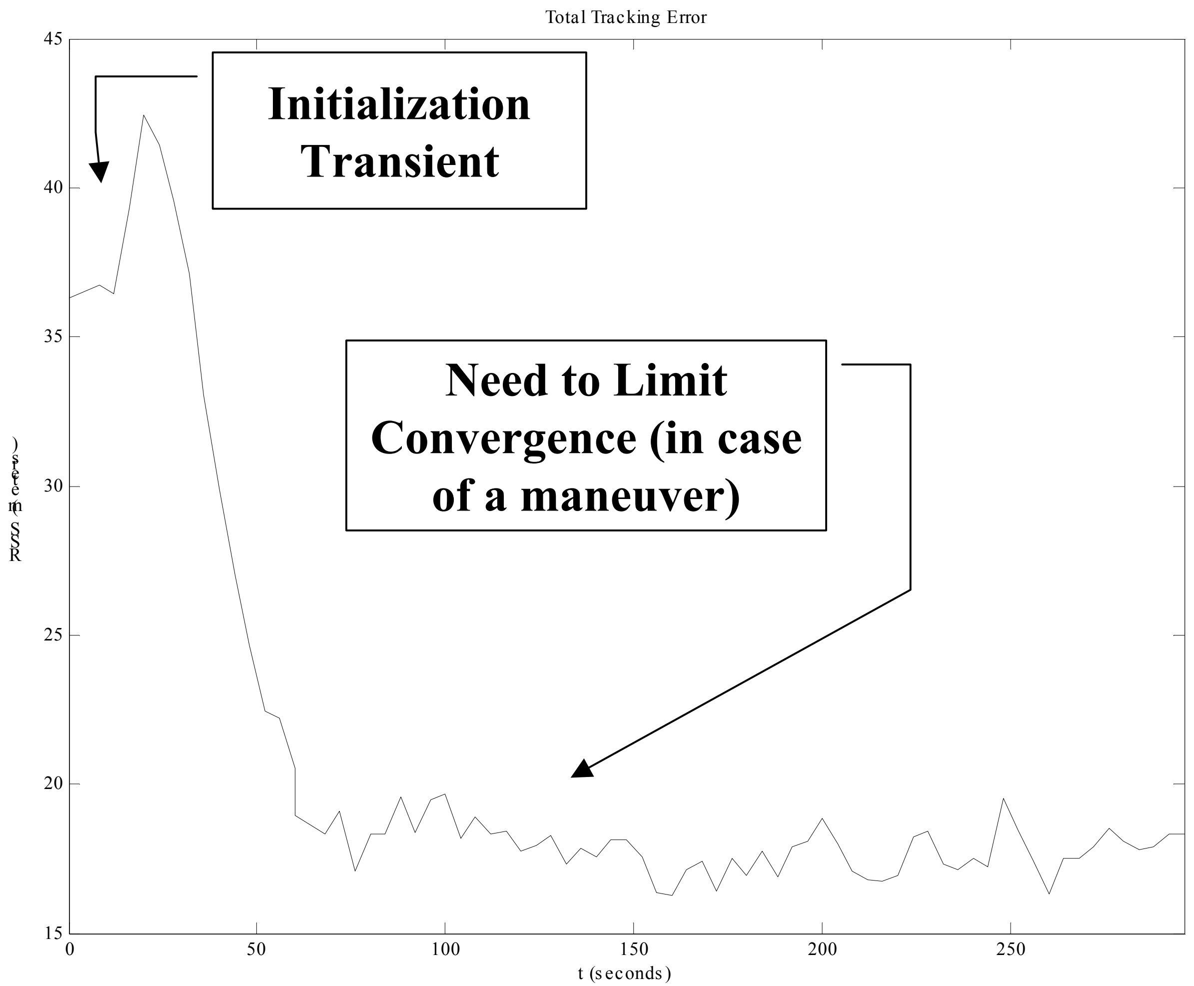
Track in Cartesian space



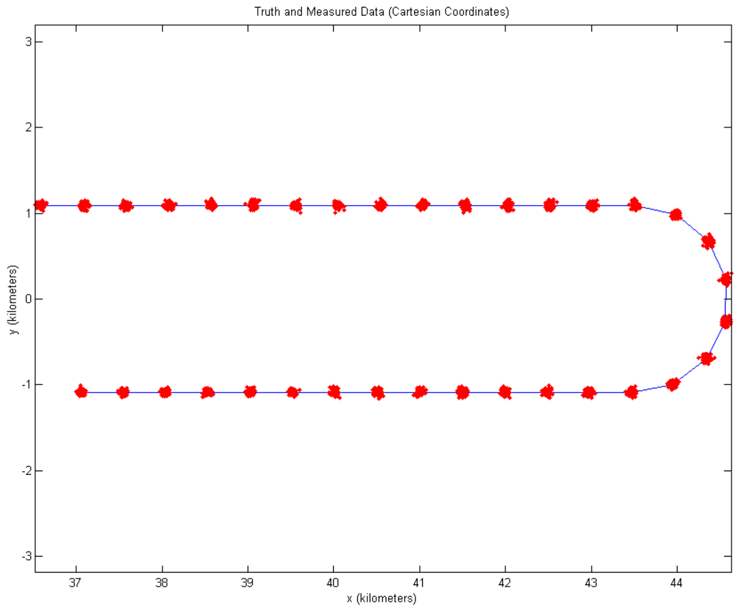
Track in Radar space

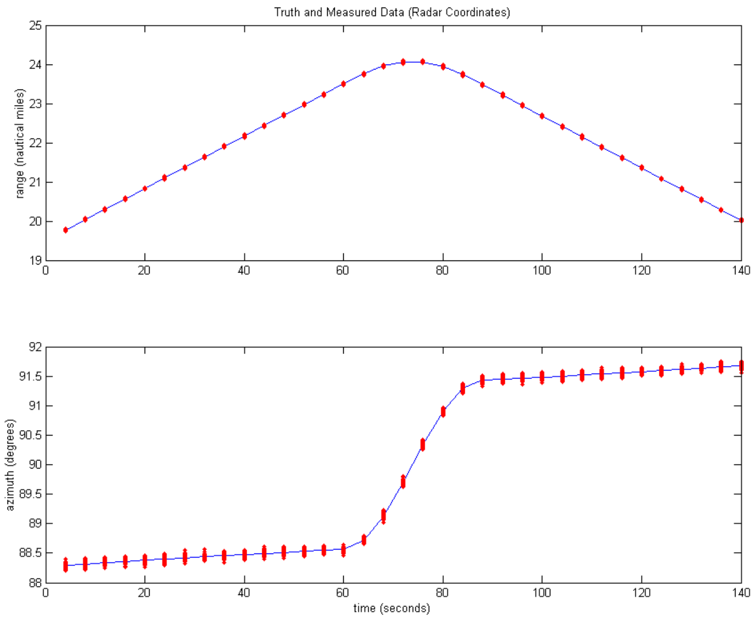


Convergence (No Maneuver)

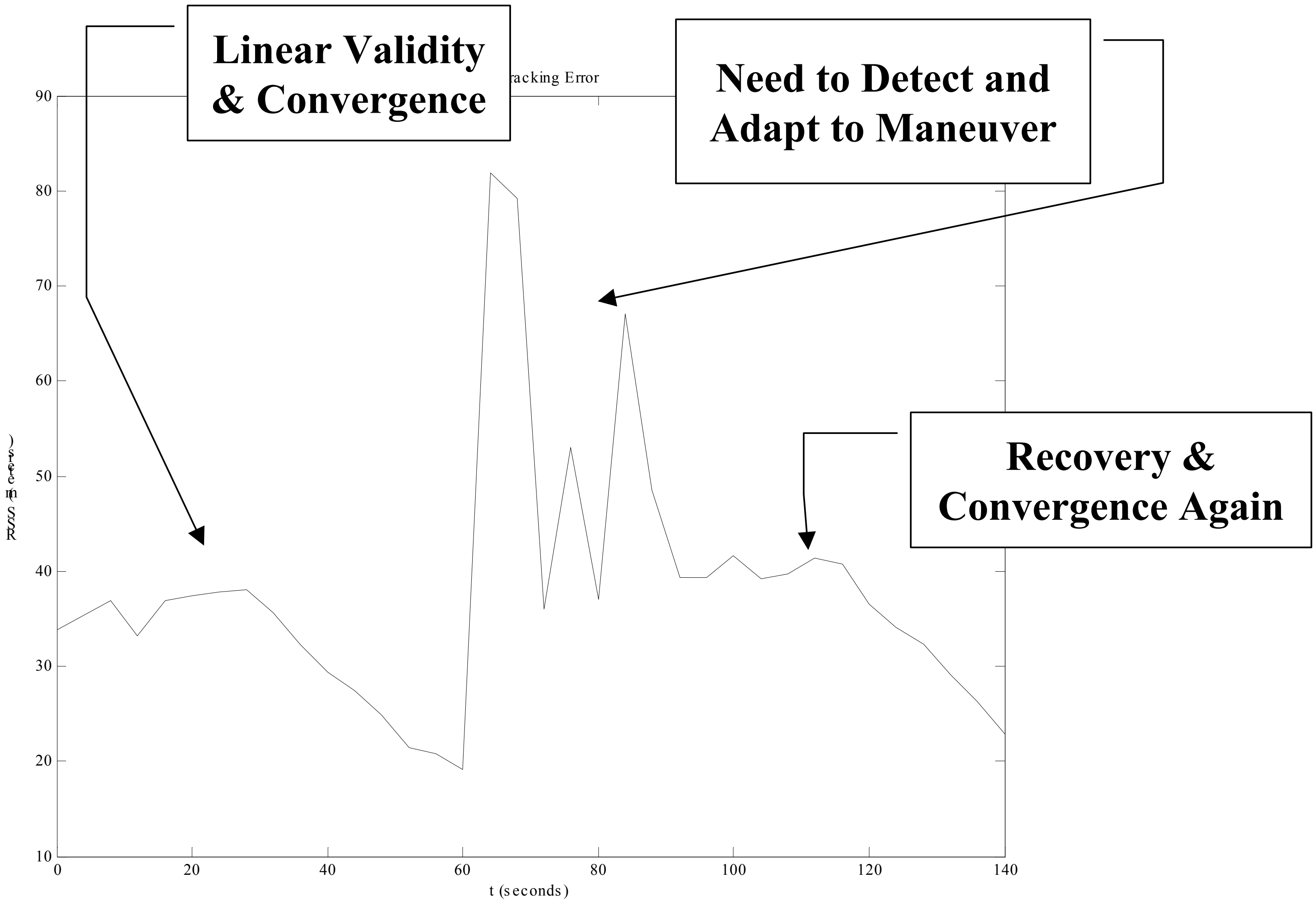


Maneuvering Example

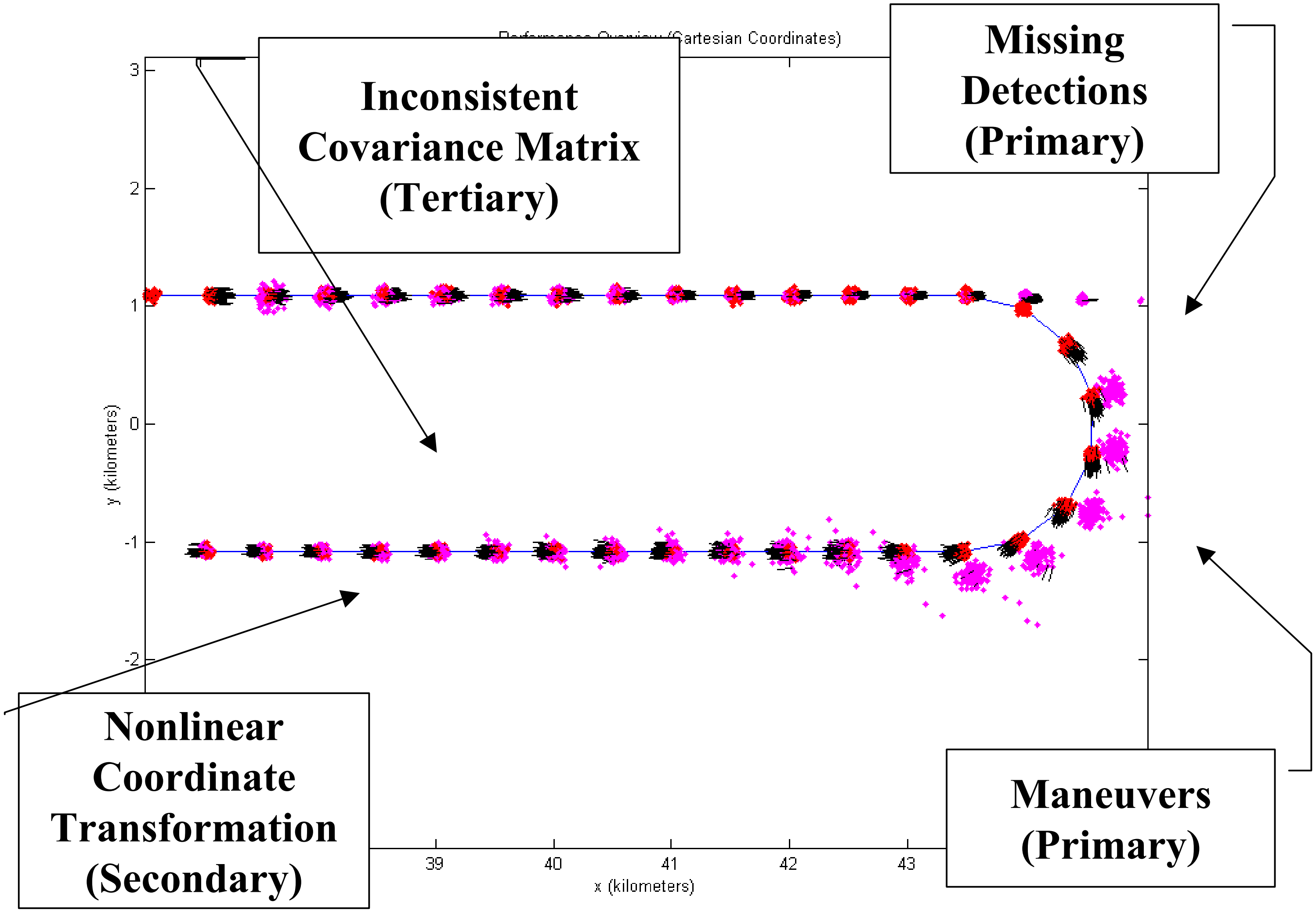




Total Tracking Error (Maneuver Case)



Effects of Nonlinearity



The Coordinate Transformation Effect

Given Two Independent Measurements

$$\left(R_i, A_i\right) \text{ and } \left(R_v, A_v\right) \text{ with } \Delta T = t_i - t_v$$

Estimate the State Vector in Radar Coordinates

$$\begin{bmatrix} \hat{r} \\ \hat{\dot{r}} \\ \hat{a} \\ \hat{\dot{a}} \end{bmatrix} = \begin{bmatrix} \left(R_i + R_v\right)/2 \\ \left(R_i - R_v\right)/\Delta T \\ \left(A_i + A_v\right)/2 \\ \left(A_i - A_v\right)/\Delta T \end{bmatrix}$$

Estimated State Vector in Cartesian Coordinates

Method 1

Transform the measurements

$$\begin{bmatrix} Y_i \\ X_i \end{bmatrix} = \begin{bmatrix} R_i \cos A_i \\ R_i \sin A_i \end{bmatrix} \text{ and } \begin{bmatrix} Y_v \\ X_v \end{bmatrix} = \begin{bmatrix} R_v \cos A_v \\ R_v \sin A_v \end{bmatrix}$$

Then estimate the state vector

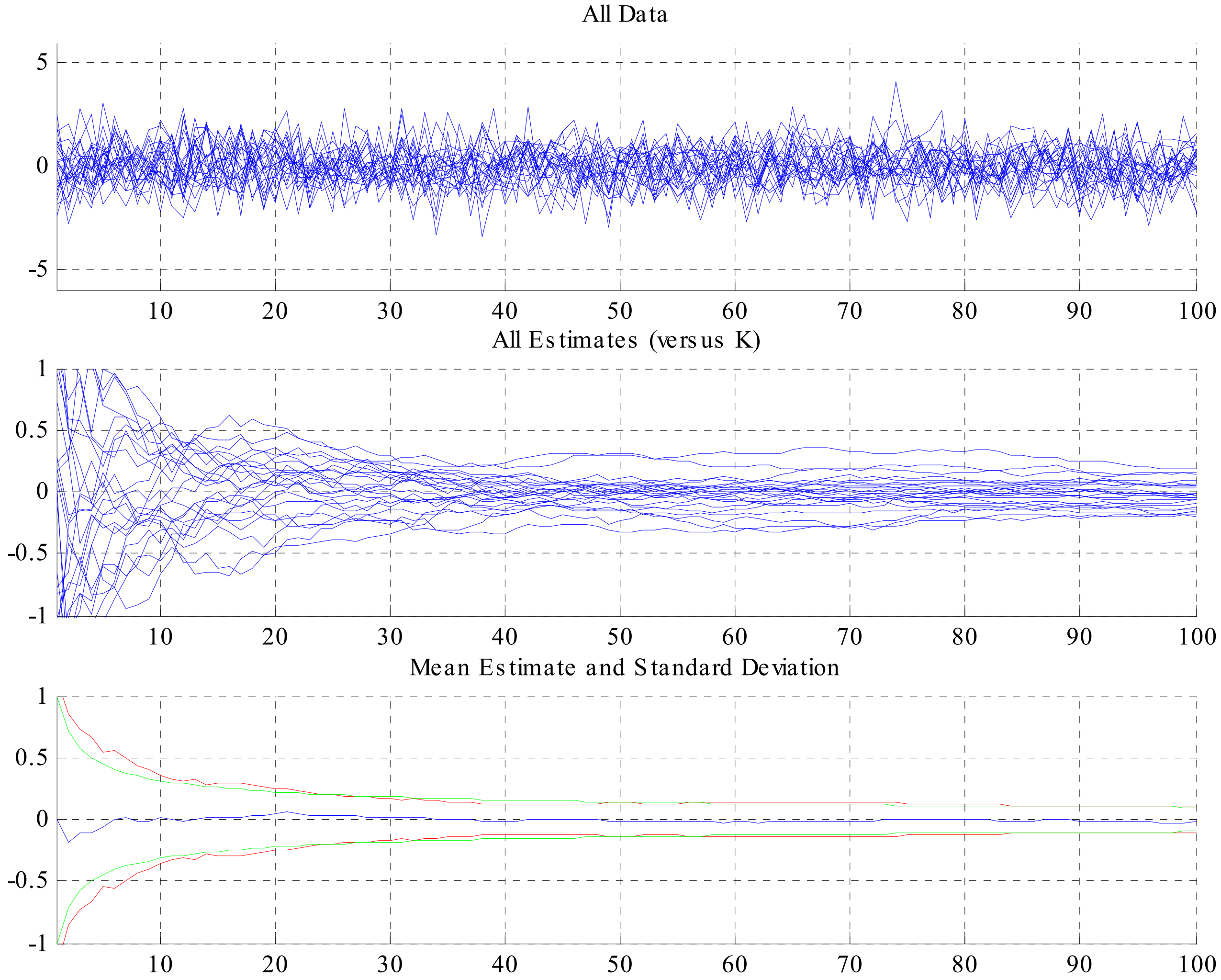
$$\begin{bmatrix} \hat{x} \\ \hat{\dot{x}} \\ \hat{y} \\ \hat{\dot{y}} \end{bmatrix} = \begin{bmatrix} \left(X_i + X_v\right)/2 \\ \left(X_i - X_v\right)/\Delta T \\ \left(Y_i + Y_v\right)/2 \\ \left(Y_i - Y_v\right)/\Delta T \end{bmatrix}$$

Method 2

Transform the estimated state vector

$$\begin{bmatrix} \hat{y}' \\ \hat{x}' \end{bmatrix} = \begin{bmatrix} \hat{r} \cos \hat{a} \\ \hat{r} \sin \hat{a} \end{bmatrix} \text{ and } \begin{bmatrix} \hat{\dot{y}}' \\ \hat{\dot{x}}' \end{bmatrix} = \begin{bmatrix} \cos \hat{a} & -\sin \hat{a} \\ \sin \hat{a} & \cos \hat{a} \end{bmatrix} \begin{bmatrix} \hat{\dot{r}} \\ \hat{r}\hat{\dot{a}} \end{bmatrix}$$

No Linearization Errors (25 Realizations)



Commutativity (Linear Case) – 1

- Given a prediction and two measurements (scalars or vectors)
  - All unbiased
  - All statistically independent
- The BLUE (as a Kalman smoother)

$$\hat{\mathbf{z}} = \mathbf{M}\left[\mathbf{M}^{-1}(-)\hat{\mathbf{z}}(-) + \mathbf{P}^{-1}\mathbf{H}^T\hat{\mathbf{x}} + \mathbf{Q}^{-1}\mathbf{G}^T\hat{\mathbf{y}}\right]$$

$$\mathbf{M}^{-1} = \mathbf{M}^{-1}(-) + \mathbf{H}^T\mathbf{P}^{-1}\mathbf{H} + \mathbf{G}^T\mathbf{Q}^{-1}\mathbf{G}$$

- The BLUE (as a linear operator)

$$\left(\hat{\mathbf{z}},\mathbf{M}^{-1}\right)_{\mathbf{I}} = \left[\left(\hat{\mathbf{z}}(-),\mathbf{M}^{-1}(-)\right)_{\mathbf{I}};\left(\hat{\mathbf{x}},\mathbf{P}^{-1}\right)_{\mathbf{H}};\left(\hat{\mathbf{y}},\mathbf{Q}^{-1}\right)_{\mathbf{G}}\right]$$

Commutativity (Linear Case) – 2

- BLUE (recursive form)
  - First update with one

$$\left(\hat{\mathbf{z}}_{\mathbf{x}},\mathbf{M}_{\mathbf{x}}^{-1}\right)_{\mathbf{I}} = \left[\left(\hat{\mathbf{z}}(-),\mathbf{M}^{-1}(-)\right)_{\mathbf{I}};\left(\hat{\mathbf{x}},\mathbf{P}^{-1}\right)_{\mathbf{H}}\right] \Rightarrow$$

$$\left(\hat{\mathbf{z}},\mathbf{M}^{-1}\right)_{\mathbf{I}} = \left[\left(\hat{\mathbf{z}}_{\mathbf{x}},\mathbf{M}_{\mathbf{x}}^{-1}\right)_{\mathbf{I}};\left(\hat{\mathbf{y}},\mathbf{Q}^{-1}\right)_{\mathbf{G}}\right]$$

$$\left(\hat{\mathbf{z}}_{\mathbf{y}},\mathbf{M}_{\mathbf{y}}^{-1}\right)_{\mathbf{I}} = \left[\left(\hat{\mathbf{z}}(-),\mathbf{M}^{-1}(-)\right)_{\mathbf{I}};\left(\hat{\mathbf{y}},\mathbf{Q}^{-1}\right)_{\mathbf{G}}\right] \Rightarrow$$

$$\left(\hat{\mathbf{z}},\mathbf{M}^{-1}\right)_{\mathbf{I}} = \left[\left(\hat{\mathbf{z}}_{\mathbf{y}},\mathbf{M}_{\mathbf{y}}^{-1}\right)_{\mathbf{I}};\left(\hat{\mathbf{x}},\mathbf{P}^{-1}\right)_{\mathbf{H}}\right]$$

  - Then update with the other

$$\left(\hat{\mathbf{z}},\mathbf{M}^{-1}\right)_{\mathbf{I}} = \left\{\left[\left(\hat{\mathbf{z}}(-),\mathbf{M}^{-1}(-)\right)_{\mathbf{I}};\left(\hat{\mathbf{x}},\mathbf{P}^{-1}\right)_{\mathbf{H}}\right];\left(\hat{\mathbf{y}},\mathbf{Q}^{-1}\right)_{\mathbf{G}}\right\}$$

$$\left(\hat{\mathbf{z}},\mathbf{M}^{-1}\right)_{\mathbf{I}} = \left\{\left[\left(\hat{\mathbf{z}}(-),\mathbf{M}^{-1}(-)\right)_{\mathbf{I}};\left(\hat{\mathbf{y}},\mathbf{Q}^{-1}\right)_{\mathbf{G}}\right];\left(\hat{\mathbf{x}},\mathbf{P}^{-1}\right)_{\mathbf{H}}\right\}$$

Commutativity (Linear Case) – 3

- The BLUE as a Filter
  - The smoother form

$$\hat{\mathbf{z}}_{\mathbf{x}} = \mathbf{M}\left[\mathbf{M}^{-1}(-)\hat{\mathbf{z}}(-) + \mathbf{P}^{-1}\mathbf{H}^{T}\hat{\mathbf{x}}\right] \text{ and } \mathbf{M}_{\mathbf{x}}^{-1} = \mathbf{M}^{-1}(-) + \mathbf{H}^{T}\mathbf{P}^{-1}\mathbf{H}$$

  - The filter form

$$\begin{aligned}\hat{\mathbf{z}}_{\mathbf{x}} &= \hat{\mathbf{z}}(-) + \mathbf{M}_{\mathbf{x}}\mathbf{H}^{T}\mathbf{P}^{-1}\left[\hat{\mathbf{x}} - \mathbf{H}\hat{\mathbf{z}}(-)\right] \text{ and } \mathbf{M}_{\mathbf{x}} = \left(\mathbf{I} - \mathbf{K}_{\mathbf{x}}\mathbf{H}\right)\mathbf{M}(-) \\ &= \hat{\mathbf{z}}(-) + \mathbf{K}_{\mathbf{x}}\left[\hat{\mathbf{x}} - \mathbf{H}\hat{\mathbf{z}}(-)\right]\end{aligned}$$

$$\mathbf{K}_{\mathbf{x}} = \mathbf{M}(-)\mathbf{H}^{T}\left[\mathbf{H}\mathbf{M}(-)\mathbf{H}^{T} + \mathbf{P}^{-1}\right]^{-1}$$

The Non-Linear Case (Batch) – 1

- The coordinate transformation

$$r = \sqrt{x^2 + y^2} \quad \text{and} \quad a = \arctan\left(y, x \right)$$

$$y = r\cos a \quad \text{and} \quad x = r\sin a$$

- The Measurement Model

$$\begin{bmatrix} R \\ A \end{bmatrix} = \begin{bmatrix} r(y,x) \\ a(y,x) \end{bmatrix} + \begin{bmatrix} n_R \\ n_A \end{bmatrix} \quad \text{and} \quad \mathbf{R} = \begin{bmatrix} \sigma_R^2 & 0 \\ 0 & \sigma_A^2 \end{bmatrix}$$

- The linearized BLUE (Extended Kalman Filter)
  - State estimate update

$$\hat{\mathbf{x}} = \hat{\mathbf{x}}(-) + \mathbf{P}\left(\frac{d}{d\mathbf{x}^T} \begin{bmatrix} r\left(\hat{\mathbf{x}}(-)\right) \\ a\left(\hat{\mathbf{x}}(-)\right) \end{bmatrix} \right) \mathbf{R}^{-1} \left(\begin{bmatrix} R \\ A \end{bmatrix} - \begin{bmatrix} r\left(\hat{\mathbf{x}}(-)\right) \\ a\left(\hat{\mathbf{x}}(-)\right) \end{bmatrix} \right)$$

    - Note: $\operatorname{var} \hat{\mathbf{x}} \equiv \mathbf{\Sigma}_{\mathbf{x}} \neq \mathbf{P}$

The Non- Linear Case (Batch) – 2

- The EKF State Vector Update is essentially a differential

$$\mathbf{P}^{-1}\left(\hat{\mathbf{x}}-\hat{\mathbf{x}}(-)\right)=\left(\frac{d}{d\mathbf{x}^T}\begin{bmatrix} r\left(\hat{\mathbf{x}}(-)\right) \\ a\left(\hat{\mathbf{x}}(-)\right) \end{bmatrix}\right)\mathbf{R}^{-1}\left(\begin{bmatrix} R \\ A \end{bmatrix}-\begin{bmatrix} r\left(\hat{\mathbf{x}}(-)\right) \\ a\left(\hat{\mathbf{x}}(-)\right) \end{bmatrix}\right)$$

  - The total derivative of the measurement model is

$$\begin{bmatrix} r \\ \dot{a} \end{bmatrix}=\mathbf{J}\left(1/r\right)\mathbf{U}\left(a\right)\begin{bmatrix} \dot{y} \\ \dot{x} \end{bmatrix}\Rightarrow\frac{d}{d\mathbf{x}^T}\begin{bmatrix} r(\mathbf{x}) \\ a(\mathbf{x}) \end{bmatrix}=\mathbf{U}^T\left(a\right)\mathbf{J}\left(r\right)$$

    - where

$$\mathbf{U}^T\left(a\right)=\begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix} \text{ and } \mathbf{J}\left(r\right)=\begin{bmatrix} 1 & 0 \\ 0 & r \end{bmatrix}$$

Non-Linear Case (Sequential *R* then *A*)

- First update with range (linearize with respect to the more accurate measurement first)

$$\hat{\mathbf{x}}_R = \hat{\mathbf{x}}(-) + \mathbf{K}_R\left[R - r\left(\hat{\mathbf{x}}(-)\right)\right] \text{ and } \mathbf{P}_R = \left[\mathbf{I} - \mathbf{K}_R\mathbf{h}_R^T\right]\mathbf{P}(-)$$

  – With

$$\mathbf{K}_R = \mathbf{P}(-)\mathbf{h}_R^T \Big/ \left[\mathbf{h}_R\mathbf{P}(-)\mathbf{h}_R^T + \sigma_R^2\right] \text{ and } \mathbf{h}_R \equiv \frac{dr\left(\hat{\mathbf{x}}(-)\right)}{d\mathbf{x}^T}$$

- Followed by

$$\hat{\mathbf{x}}_{RA} = \hat{\mathbf{x}}_R + \mathbf{K}_{RA}\left[A - a(\hat{\mathbf{x}}_R)\right] \text{ and } \mathbf{P}_{RA} = \left[\mathbf{I} - \mathbf{K}_{RA}\mathbf{h}_{RA}^T\right]\mathbf{P}_R$$

  – With

$$\mathbf{K}_{RA} = \mathbf{P}_R(-)\mathbf{h}_{RA}^T \Big/ \left[\mathbf{h}_{RA}\mathbf{P}_R\mathbf{h}_{RA}^T + \sigma_A^2\right] \text{ and } \mathbf{h}_{RA} \equiv \frac{da\left(\hat{\mathbf{x}}_R\right)}{d\mathbf{x}^T}$$

- Note: $\hat{\mathbf{x}}_{RA} \approx \hat{\mathbf{x}}$ and $\mathbf{P}_{RA} \approx \mathbf{P}$

Non-Linear Case (Sequential *A* then *R*)

- First update with range (linearize with respect to the more accurate measurement first)

$$\hat{\mathbf{x}}_A = \hat{\mathbf{x}}(-) + \mathbf{K}_A\left[A - a\left(\hat{\mathbf{x}}(-)\right)\right] \text{ and } \mathbf{P}_A = \left[\mathbf{I} - \mathbf{K}_A\mathbf{h}_A^T\right]\mathbf{P}(-)$$

  – With

$$\mathbf{K}_A = \mathbf{P}(-)\mathbf{h}_A^T \Big/ \left[\mathbf{h}_A\mathbf{P}(-)\mathbf{h}_A^T + \sigma_A^2\right] \text{ and } \mathbf{h}_A \equiv \frac{da\left(\hat{\mathbf{x}}(-)\right)}{d\mathbf{x}^T}$$

- Followed by

$$\hat{\mathbf{x}}_{AR} = \hat{\mathbf{x}}_A + \mathbf{K}_{AR}\left[R - r(\hat{\mathbf{x}}_A)\right] \text{ and } \mathbf{P}_{AR} = \left[\mathbf{I} - \mathbf{K}_{AR}\mathbf{h}_{AR}^T\right]\mathbf{P}_A$$

  – With

$$\mathbf{K}_{AR} = \mathbf{P}_A(-)\mathbf{h}_{AR}^T \Big/ \left[\mathbf{h}_{AR}\mathbf{P}_A\mathbf{h}_{AR}^T + \sigma_R^2\right] \text{ and } \mathbf{h}_{AR} \equiv \frac{dr\left(\hat{\mathbf{x}}_A\right)}{d\mathbf{x}^T}$$

- Note: $\hat{\mathbf{x}}_{AR} \approx \mathbf{x}$ and $\mathbf{P}_{AR} \approx \mathbf{P}_{RA} \approx \mathbf{P} \approx \operatorname{var}\hat{\mathbf{x}}_{AR}$

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