Clutter Processing Using K-Distributions for Digital Radars with Increased Sensitivity

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Abstract - This paper presents a unique false alarm mitigation approach for nonhomogeneous clutter, which is problematic for digital radars with increased sensitivity. A clutter map is formed containing estimates of the two parameters for the K-distribution. The map applies the new thresholds to the data. The false alarm rate is reduced by a factor of 1000 due to the improved accuracy in modeling the clutter distribution tail.

I. INTRODUCTION

In this paper, we present a new clutter processing approach that successfully mitigates false alarms by modeling clutter with K-distributions. The addition of digital processing to radars today increases the radar's sensitivity. This additional sensitivity requires more robust clutter processing to maintain optimal system performance. Non-homogeneous, spiky clutter is particularly problematic. The processing presented in this paper controls clutter false alarms for this type of clutter using a clutter map based on the K-distribution. The paper demonstrates the effectiveness of this processing using real radar data collected with a highly digitized, ground surveillance radar.

Clutter maps are designed to handle nonhomogeneous clutter, which leaks through most traditional false alarm processing. CFAR processing is designed for clutter that is homogeneous in range. Adaptive detection techniques also rely on homogeneity in range and sets thresholds using secondary data collected from neighboring range cells [1][2][3][4][5]. This paper proposes a false alarm mitigation approach that accurately models the clutter using K-distributions for each cell [6][7][8]. A map is created with the higher order statistics generated from collected data measurements. From these moments, K-distribution parameters are estimated. A new threshold to be applied to the radar measurements is computed for the appropriate K-distribution. Since this approach independently adapts to the clutter in each cell, the clutter map thresholds are effective in reducing false alarms while maintaining detection performance in nonhomogeneous clutter.

This paper is subdivided into five sections including the introduction. The second section of this paper describes the clutter models considered for the clutter map. The third section presents the data and a few statistical moment estimates. The fourth section describes the proposed clutter map approach. The final section describes our conclusions and suggests further work in this area.

II. THE CLUTTER MODELS

Clutter map performance depends upon the accuracy of the clutter model. A generally accepted model for the clutter return variations is the Rayleigh distribution. This distribution is not accurate for every type of clutter. While many other distributions have been proposed, the K-distribution models spiky terrain clutter very well[8]. This section reviews the Rayleigh distribution and the K-distribution.

A Rayleigh distribution is completely described by a single parameter, its mean value. The Rayleigh probability density function (pdf) is

$$f_x(x) = \frac{2x}{b} e^{\frac{-x^2}{b}} . \quad (1)$$

The parameter, b, can be estimated from the sample mean of the data, $\hat{\mu}$, using

$$b = \frac{4}{\pi}\hat{\mu}^2 . \quad (2)$$

In contrast, the K-distribution models requires two parameters to be completely defined. Its pdf is

$$K_x = \frac{2}{a\Gamma(\nu + 1)}\left(\frac{x}{2a}\right)^{\nu + 1} \kappa_\nu\left(\frac{x}{a}\right) U(x) \quad (3)$$

where a is a scaling factor, ν is the shape factor, $\Gamma(\ldots)$ is the gamma function, and $\kappa_\nu(\ldots)$ is the modified Bessel function of the second kind of order ν [7][8]. The two basic parameters, ν, and a can be estimated using sample estimates of the moments. There are more optimum techniques of estimating the K-distribution parameters but these are computationally intensive and not practical for real-time implementation in a clutter map. A suboptimal approach is used for this clutter map and described later in this paper.

The shape parameter, ν, controls the “spikyness” of the distribution and must be greater than -1. Figure 1 contains plots of various K-distributions with a set at 15. The tail lengthens as ν decreases. The K-distribution can also be equivalent to the exponential, Rayleigh, or Weibull distributions with the appropriate parameter values. Figure 2 contains two identical curves for a K, a ν of -.5 and an a equal to 30, and exponential distribution, a mean of 30. Figure 3 also contains a plot of a Rayleigh pdf with a mean of 30 and a K-distribution with ν of 300 and an a of 1. As ν approaches infinity the distribution appears more like a Rayleigh distribution.

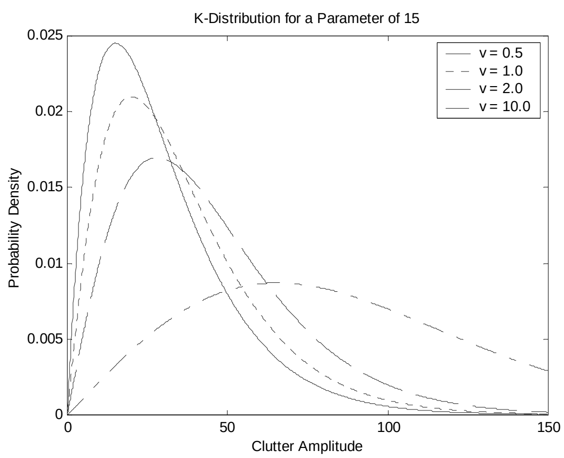

Fig. 1.K-Distribution for Several Shape Parameters

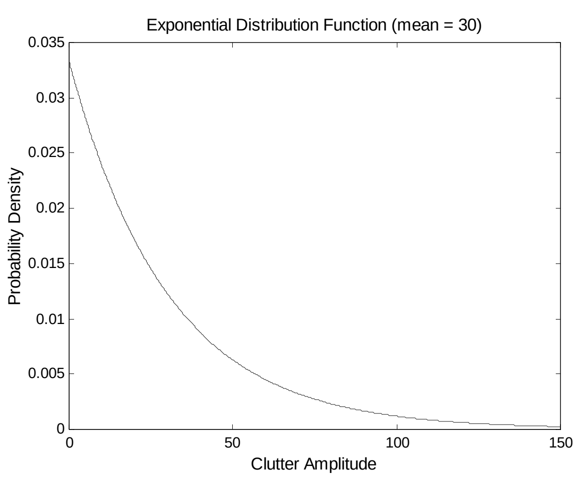

Fig. 2.Exponential Distribution and Equivalent K-Distribution

III. Data Collection and Analysis

The data for this study is collected using a horizontally polarized, ground surveillance radar. The radar transmits an LFM signal which is received and processed by a digital signal processor. The data is recorded before any clutter processing such as CFAR or MTI is performed. Since our processing approach requires estimates of the higher order and fractional moment from the data samples, an assumption is made that the clutter is ergodic and stationary over the observation interval, approximately 15 minutes. The data used in this study is from particularly spiky, non-homogenous clutter. The data is collected from patches of clutter covering eight contiguous range cells in 4-5 neighboring azimuth beams.

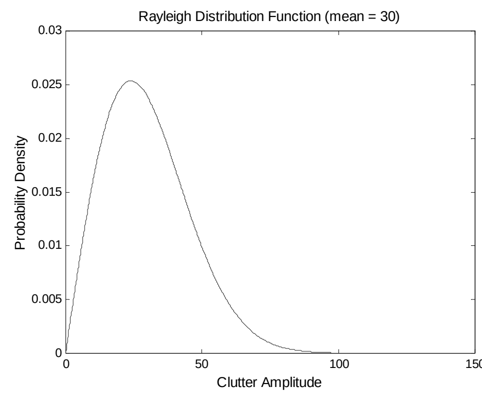

Fig. 3.Rayleigh Distribution and Equivalent K-Distribution

Once enough data is collected, sample estimates of the clutter’s central moments are formed. The sample mean, an unbiased estimates of the mean, is computed from

$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad (4)$$

where x_i is the amplitude of the data sample, i is the sample index, and N is the number of data points collected during the observation interval for one range cell. The sample variance is estimated using

$$\hat{\sigma} = \frac{1}{(N-1)}\sum_{i=1}^{N} (x_i - \hat{\mu})^2 . \qquad (5)$$

The normalized skew, indicating the asymmetry of the distribution tails, is estimated by

$$\hat{\zeta} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{x_i - \hat{\mu}}{\hat{\sigma}}\right)^3 . \quad (6)$$

The kurtosis is the fourth central moment and gives an indication of sharpness of the density function's peak. The Fisher kurtosis, which sets the Gaussian kurtosis to 0, compares the clutter distribution's peak sharpness with that of a Gaussian distribution and is computed from

$$\gamma = \left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{x_i - \hat{\mu}}{\hat{\sigma}}\right)^4\right] - 3 . \quad (7)$$

These estimated moments support a quick characterization of the clutter. The skew and kurtosis are normalized by the estimated standard deviation and indicate the rough shape of the pdf. For a Gaussian distribution, the skew and kurtosis are 0. The Rayleigh distribution has a skew of .6 which reflects the assymetry of its distribution tails. The kurtosis is .2 indicating a distribution that is more peaked near the mean. Table I summarizes the central moments for a variety of distributions including some K-distributions. Notice that the last K-distribution's 4 moments are identical to a Rayleigh distribution. As the shape parameter increases, the skew and kurtosis decreases for a K-distribution indicating a less spiky distribution.

TABLE I.
DISTRIBUTIONS AND CENTRAL MOMENTS

Distribution	mean	variance	skew	kurtosis
Gaussian	a	σ^2	0	0
Rayleigh	a	$\left(\frac{a}{2}\right)^2$	0.6	0.2
Exponential	$\frac{1}{a}$	$\left(\frac{1}{a}\right)^2$	2	6
K: $\nu = 0.5$ $a = 15$	30	$(21)^2$	1.4	2.8
K: $\nu = 1$ $a = 15$	28	$(18)^2$	1.2	2.2
K: $\nu = 2$ $a = 15$	34	$(21)^2$	1.1	1.5

TABLE I.
DISTRIBUTIONS AND CENTRAL MOMENTS

Distribution	mean	variance	skew	kurtosis
K: $\nu = 10$ $a = 15$	63	$(35)^2$	0.8	0.6
K: $\nu = 600$ $a = 15$	30	$(15)^2$	0.6	0.2

As an example in Figure 4 , the four moments are computed and plotted for 2 frequencies and 8 contiguous cells for one azimuth beam and 80 samples. One notices immediately that this clutter is not homogeneous nor is it Rayleigh distributed. Also, one notices that the statistics are consistent between the two frequencies so that the clutter model may be selected independent of frequency. However, the parameters will be slightly different for each frequency so a separate map must be maintained for each frequency.

In low level clutter signal returns where the system noise dominates, a negative kurtosis may result indicating more spread than either a Rayleigh or Gaussian distribution. The clutter map should not be applied in these regions and the normal thresholds should be used. A K-distribution model works well for statistics illustrated in Figure 4 for the cells with estimated means above 25.

IV. PROPOSED CLUTTER MAP

The proposed clutter map handles this nonhomogeneous clutter by adapting the clutter model using estimated statistical moments from collected data. The K-distribution can properly model the spiky clutter with longer distribution tails as well as approximating the Weibull, exponential, or Rayleigh distribution. The two remaining issues to this clutter map processing are the computational complexity of the technique and the number of samples required for the estimate to converge. In this section, three approaches to computing the scaling and shape factor are described with differing convergence properties.

One technique with minimal complexity computes the two parameters for the K-distribution from high order sample moments using (8), (9), and (10). This technique is straight forward and does not require the solution to a system of nonlinear equations. The scaling and shape factor are computed by

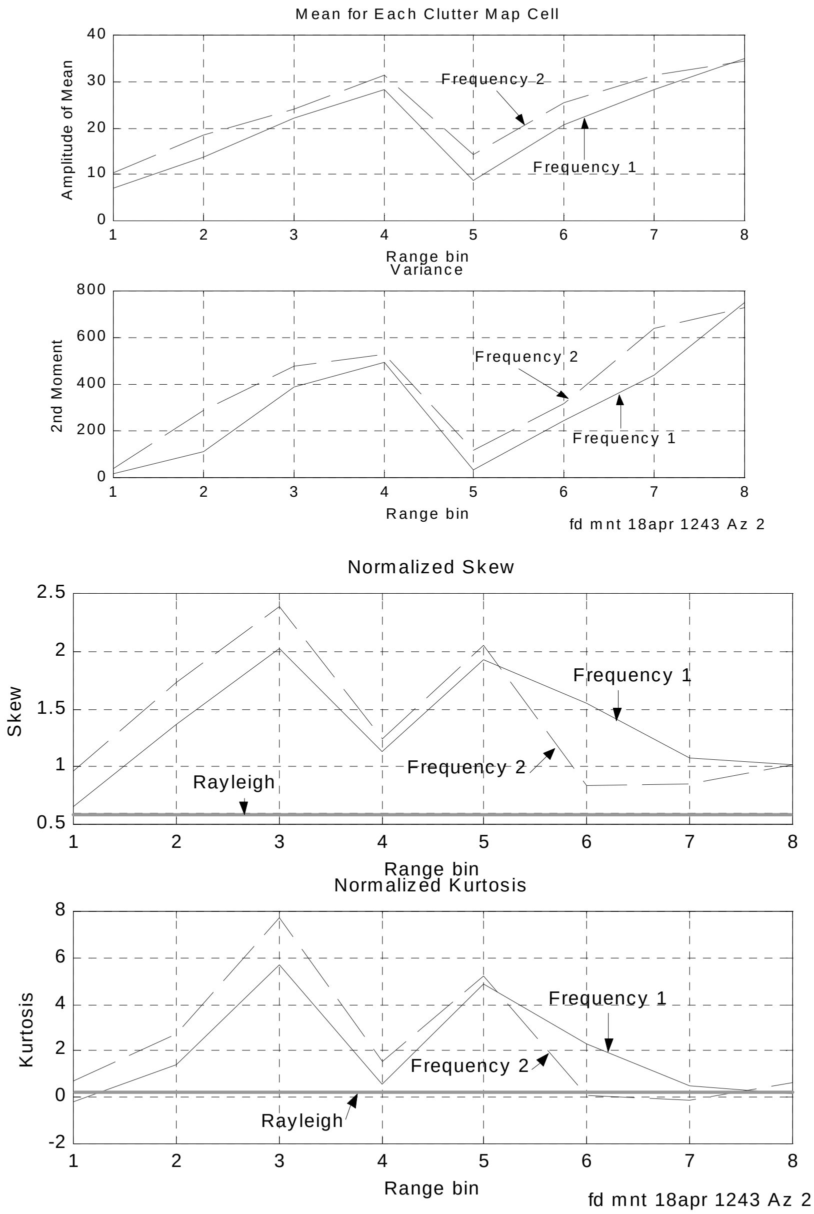

Fig. 4.Statistics For 8 Contiguous Range Cells in One Azimuth Beam

$$\nu_{high} = \frac{4 - \frac{\mu_4}{\mu_2^2}}{\frac{\mu_4}{\mu_2^2} - 2} \quad \text{(8)}$$

and

$$a = \frac{\mu_1 \Gamma(\nu + 1)}{\sqrt{\pi}\Gamma(\nu + 1.5)} \quad \text{(9)}$$

where μ_k is the sample estimate of the kth moment[7]. The sample estimate of the moments is computed from

$$\mu_k = \frac{1}{N}\sum_{i=1}^{N} x_i^k \quad \text{(10)}$$

where N is the number of samples collected.

The K-distribution parameters can also be estimated from low-order and fractional moments[7]. The shape parameter can either be computed from

$$\nu_{low} = \frac{\frac{25}{16} - \frac{\mu_{\frac{5}{2}}}{\mu_{\frac{1}{2}}\mu_2}}{\frac{\mu_{\frac{5}{2}}}{\mu_{\frac{1}{2}}\mu_2} - \frac{5}{4}} \quad (11)$$

or

$$\nu_{med} = \frac{\frac{9}{4} - \frac{\mu_3}{\mu_1\mu_2}}{\frac{\mu_3}{\mu_1\mu_2} - \frac{3}{2}}. \quad (12)$$

Of the two equations, (11) uses the lowest order and fractional moments.

The number of samples required for the parameter estimater to converge is the primary implementation problem in any real-time radar system. The convergence properties of these K-distribution parameter estimators have been analyzed using a Monte-Carlo approach. The complexity of the estimator prohibits an analytical approach. The plot in Figure 5 gives the standard deviation of the estimated shape parameter as a function of the number of samples for the estimators in (8), (11), and (12). Equation (11) consistently gives a lower standard deviation. In Figure 6 , the effect of the clutter's spikyness on the estimator's convergence is analyzed. The standard deviation of the estimator in (11) is plotted as a function of the number of samples for 3 different shape parameter values. The spikier distributions converge more quickly. Therefore, it takes more samples to estimate less "spiky" clutter with K-distributions.

Figure 7 and Figure 8 illustrates the new distribution models resulting from this new clutter map approach. Figure 7 is a range cell with "spiky" clutter. Figure 8 has clutter that can be modelled with a Rayleigh distribution. The key to properly setting the clutter thresholds is accurately estimating the pdf tails. The "spiky" clutter requires longer tails. This results in higher thresholds and a reduction in the false alarm rate.

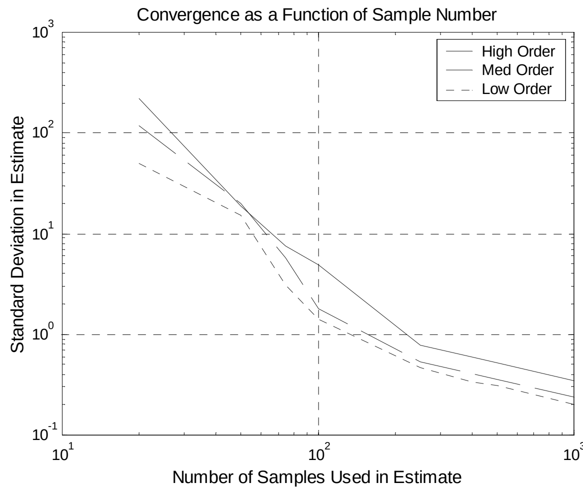

Fig. 5.Convergence Properties for Estimation Techniques

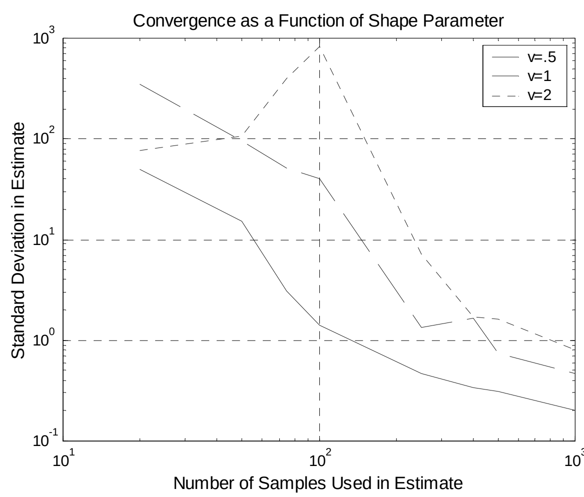

Fig. 6.Convergence Properties as a Function of the Shape Parameter

In Figure 7 , the histogram or frequency plot of very spiky clutter data collected at L-band is given. The skew of this clutter data is 1.24, and the kurtosis is 1.6. Four distributions are then estimated: a Rayleigh distribution using (2). and three K-distributions using (8), (9), (11), and (12). The best estimate of the K-distribution parameters results from (11) and (9) due to the convergence properties mentioned previously. The estimator gives a shape factor of $\nu = 42$ and scaling of $a = 16.2$ for the K-distribution model. The resulting threshold is 267 for a false alarm probability of 1e-6. If the Rayleigh clutter model had been erroneously used , the detection threshold would be set at 132 to achieve the false alarm probability of 1e-6. However, this poor modelling would have increased the false alarm rate to 2.4e-3 instead.

Figure 8 shows a frequency plot and resulting clutter models for clutter that is nearly Rayleigh. The K-distributions are converging to the Rayleigh distribution. The thresholds are nearly equivalent between the Rayleigh and the K-distribution. The K-distribution parameter estimators, however, do converge more slowly for Rayleigh clutter than spikier clutter.

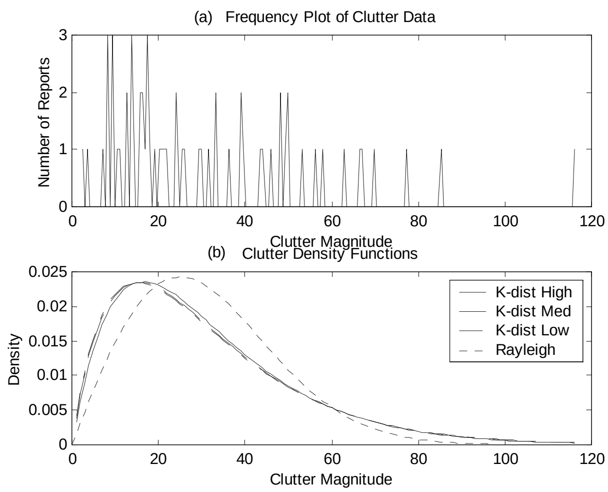

Fig. 7. Clutter Histogram and Clutter Density Functions for Spiky Clutter

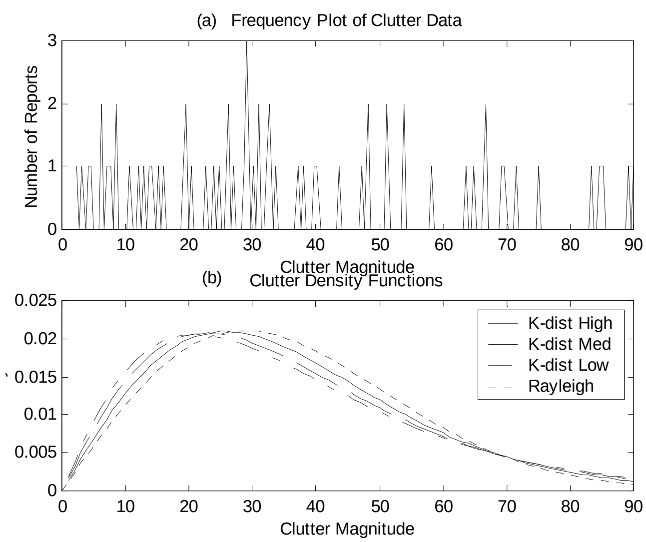

Fig. 8. Histogram and Clutter Density Functions for Rayleigh Clutter (a) actual clutter sample results (60 samples) (b) computed density functions from clutter sample moments

V. Conclusions

This paper demonstrates that a clutter map that adapts the K-distribution model to fit the clutter data can significantly reduce the false alarm rate. The Rayleigh distribution is ineffectual in setting a detection threshold to reduce false alarms for nonhomogeneous, "spiky" clutter. The K-distribution with its longer tails better fits this "spiky" clutter. The longer tail increases the detection threshold and effectively reduces false alarms.

However, it is beneficial for the clutter map to switch from the K-distribution to a Rayleigh distribution if the clutter statistics indicate the Rayleigh model. This is due to the convergence properties of the estimator. A decision to change the clutter model can be made from the sample moments. Convergence and stability of the estimators on real data is an area where more research should be done.

Implementation of this clutter map requires three moments to be estimated for each operational frequency increasing the computational complexity of the clutter map. A digital processing can be designed into the system minimizing the impact of this additional complexity in the signal processing. The fact that the false alarm performance can be improved by a factor of 1000 justifies the increased complexity of the clutter map.

VI. References

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