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Abstract—A two-tiered network model has been proposed recently for prolonging lifetime and improving scalability in wireless sensor networks. This two-tiered network is a cluster-based network. Relay nodes are placed in the playing field to act as cluster heads and to form a connected topology for data transmission in the higher tier. They are able to fuse data packets from sensor nodes in their clusters and send them to sinks through wireless multi-hop paths. However, this model is not fault-tolerant as the network may be disconnected if a relay node fails. In this paper, we formulate and study a fault-tolerant relay node placement problem in wireless sensor networks. In this problem, we want to place a minimum number of relay nodes to the playing field of a sensor network such that (1) each sensor node can communicate with at least two relay nodes and (2) the network of the relay nodes is 2-connected. We present a polynomial time approximation algorithm for this problem and prove the worst-case performance given by our algorithm is bounded within $O(D \log n)$ times of the size of an optimal solution, where $n$ is the number of sensor nodes in the network, $D$ is the $(2, 1) - \text{Diameter}$ of the network formed by a sufficient set of possible positions for relay nodes.

Index Terms: Wireless sensor network, Relay node placement, Fault-tolerance.

I. INTRODUCTION

Wireless sensors can be employed for various kinds of tasks such as environmental monitoring, battlefield surveillance, biomedical observation and so on. Wireless sensor network has received intensive research attentions due to its enormous application potentials [1]. There are three major concerns for designing efficient wireless sensor networks:

Energy efficiency: Every sensor node has very limited computing and communication capability, especially very limited energy resource. Sensor nodes are normally powered by batteries and can only last for a short period of time operating at high transmitting level. Hence, the energy efficient design is required for prolonging network lifetime.

Fault-tolerance: Sensor nodes are very vulnerable to failures. They may lose functionalities at any time because of energy depletion, harsh environment factors or malicious attack from enemies. So it is important to consider survivability in sensor network.

Scalability: Usually a sensor network is required to cover a big geographic domain. New nodes may also be added to the network at any time. Therefore sensor networks may be of very large scale and network protocols are supposed to be scalable.

How to gather data packets from sensor nodes to sinks is a basic problem in wireless sensor networks. Generally sinks are far from data sources and the communication range of the sensor node is very limited. Therefore multi-hop paths need to be established for data routing. However, participating in data routing may lead to a large volume of energy consumption. Therefore, the two-tiered network model is proposed to prolong network lifetime and improve network scalability [6], [7]. The two-tiered network is actually the cluster-based network. In the network, sensor nodes are on the lower tier. They are randomly deployed in the playing field. They are able to sense the vicinity, generate corresponding data packets and send them directly to their cluster heads. The higher tier is composed of relay nodes, they are also called gateway nodes in [6] and Application Nodes (AN) in [8]. Each relay node acts as the cluster head in the corresponding cluster. It is more powerful than sensor node in terms of energy storage, computing and communication capability. It can extract useful information and remove redundancy of data packets from all sensor nodes in its cluster. Then it generates outgoing packets with much smaller total size and sends them to sinks through multi-hop paths along the relay nodes [8].

The two-tiered network architecture is scalable since sensor nodes are grouped into clusters. Moreover, in-network data fusion is applied to decrease the traffic load and sensor nodes are not involved in data routing so the network lifetime will be prolonged. It is obvious that the relay nodes play a key role in the two-tiered network. However, relay nodes may fail because of aforementioned reasons. In this case, the whole cluster will lose functionalities. In order to support the survivability for the network, two or more relay nodes should be within a sensor node’s communication range (In the following, we will say a sensor node is covered by a relay node if this relay node is within its communication range). If one sensor node’s current cluster head is down, the sensor can switch to another head node. In addition, in order to extend the network lifetime, it is preferred that the traffic load is balanced among all relay nodes, i.e., every cluster has similar size. Making each sensor node covered by multiple relay nodes enables the load-balanced clustering.

In this paper, we consider how to place a minimum number of relay nodes in the playing field such that every sensor node can reach at least two relay nodes and there exists at least two node-disjoint paths between every pair of relay nodes in the network of the relay nodes. Based on this placement, when one relay node fails, each sensor node covered by this relay node can switch to one of its backup relay nodes and the remaining relay nodes will still be connected. We formulate this as an optimization problem called $2$-Connected Relay Node Double Cover ($2CRNDC$) problem. A polynomial time approximation algorithm is proposed to solve this problem and we prove that
the number of relay nodes given by our algorithm is bounded by $O(D \log n)$ times of the size of the optimal solution, where $n$ is the number of sensor nodes, and $D$ is the $(2, 1)$ – Diameter of the graph formed by possible positions for relay nodes, whose definition is given in Section IV. To our best knowledge, this is the first paper addressing fault-tolerant relay node placement in wireless sensor networks.

The rest of the paper is organized as follows. We discuss related work in Section II. We describe and formally define the problem and some notations in Section III. We present our approximation algorithm in Section IV. We present our simulation results in Section V. We conclude the paper in Section VI.

II. RELATED WORK

Directed diffusion, a flooding-based scheme, is presented in [2] for routing queries and gathering result packets in wireless sensor network. The authors of [4] presents an energy-efficient cluster-based protocol, LEACH, for gathering data packets. In LEACH, only a fraction of nodes become head nodes in each round. Data reports from non-head sensor nodes are aggregated at the head nodes and sent directly to the Base Station (BS). In [5], the authors present an improved scheme, called PEGASIS (Power-Efficient GAthering in Sensor Information Systems), which constructs a chain for data gathering and nodes on the chain take turn to transmit aggregated packets to the BS. Recently, people begin to seek fault-tolerant routing scheme for wireless network. Generally, disjoint paths are constructed to tolerate node/link failures. Suurballe in [14] proposes an optimal algorithm to compute link disjoint paths in the network. Srinivas and Modiano in [12] point out differences between disjoint path problem in the wireless network and that in traditional networks and give an elegant optimal algorithms for finding both node-disjoint and link-disjoint paths in the wireless network.

Several protocols and algorithms are presented recently for operating the two-tiered wireless sensor network. The authors of [7] propose a heuristic scheme for clustering sensor nodes. Their goal is to balance the traffic load among all gateway nodes. A fault-tolerant clustering scheme is proposed in [6] to detect the failure and to recover sensors from the failed gateway node. The optimization problems considered there are different from ours since they do not address how to place those gateway/relay nodes and assume each sensor node in the failed cluster has at least one backup relay node available. The authors of [8] also consider a two-tiered wireless sensor network consisting of sensor clusters deployed around strategic locations and BS whose locations are relatively flexible. They propose approaches to maximize the network lifetime by arranging BS location and inter-Application Node (same as our relay node) relaying optimally. Actually, they only study networks with BS and relay nodes and do not consider relay node coverage, i.e., the location relationship between sensor nodes and relay nodes when arranging their locations. In [9], Cheng et al. study the problem of placing minimum number of relay nodes to make the induced network topology globally connected, while assuming sensor nodes are not connected originally. They formulate it as an optimization problem called steiner minimum tree with minimum number of steiner points and bounded edge length, which is first proposed by Lin and Xue in [10]. Two constant bound polynomial time approximation algorithms are proposed to solve this problem.

Sensing coverage is also a very important issue in wireless sensor networks. The authors of [11] propose an approximation algorithm to compute a connected sensor cover which includes minimum number of sensor nodes, which form a connected topology and cover the given region.

The problem we are studying is somehow related to the Minimum Geometric Disk Cover problem. It is shown to be NP-hard in [16] and a polynomial time approximation scheme is given in [15].

III. PROBLEM STATEMENTS

In this section, we present a formal definition of the $2$-Connected Relay Node Double Cover Problem (2CRNDC).

Consider a sensor network consisting of randomly distributed sensor nodes in a region. We assume that all sensor nodes have the same communication range $r$. Define a sensor communication graph, $G_S = (S, E_S)$, where $S$ is the set of all sensor nodes, and two sensor nodes are connected by an edge in $E_S$ if and only if the distance between them is no more than $r$. Since sensor nodes are densely distributed, we assume that $G_S$ is $2$-connected. For the ease of our description, we say the sensor node set $S$ is $2$-connected if $G_S$ is $2$-connected.

In order to gather data from the sensor nodes, as a part of two-tiered network architecture, a set of relay nodes is added into the sensor network. A relay node can communicate with any sensor node within a distance of $r$, the communication range of the sensor nodes. We say a sensor node is covered by a relay node, if the relay node is within its communication range, $r$. Each relay node also has a communication range, by which it can communicate with other relay nodes. Again, we assume all relay nodes have the same communication range, $R$. Normally, $R$ is much bigger than $r$. In this paper, we assume that $R \geq 2r$.

To accomplish the task of gathering data, an intuitive objective would be to add a minimum number of relay nodes so that each sensor node is covered by some relay nodes. Since the gathered data reports also need to be sent to the sinks, we also require all the relay nodes to be able to communicate with one another through a multi-hop path. Furthermore, we take failure of relay nodes into account, i.e., after the failure of one relay node, each sensor node should still be covered by some relay node, and any two relay nodes should still be able to communicate with each other through a multi-hop path.

We call a set of relay nodes $2$-connected if they can still communicate with one another through a multi-hop path of relay nodes, after the failure of one of them. Now, we are ready to give the definition of the problem.

Definition. 2-Connected Relay Node Double Cover Problem (2CRNDC): Given a $2$-connected set of randomly distributed sensor nodes $S$, the communication range of sensor nodes $r$, and the communication range of relay nodes $R$, Find minimum number of locations to place the relay nodes, so that each sensor node is covered by at least two relay nodes, and the set of relay nodes is $2$-connected.
Remark: Since the failure of a relay node is usually caused by location-related factors, we make the assumption that no two relay nodes should be placed at the same location.

If we don’t consider the 2-connectivity of the set of relay nodes, then the problem becomes the Relay Node Double Cover Problem.

It is well known that the Minimum Geometric Disk Cover problem is NP-complete [16]. Hence, we conjecture that our 2-Connected Relay Node Double Cover problem is also NP-complete.

IV. THE PROPOSED ALGORITHM

In this section, we present an approximation algorithm for the 2CRNDC problem, prove its correctness, and give a bound on its performance ratio.

We call a position \( p \) a possible position of relay nodes, if there exist two sensor nodes, \( s \) and \( s' \), such that \( \text{distance}(s, p) = \text{distance}(s', p) = r \), where \( r \) is the communication range of the sensor nodes. As one can see, for any pair of sensor nodes with distance less than \( 2r \), there are two possible positions of relay node. If we ignore connectivity, one can easily verify that it is sufficient to place relay nodes at possible positions only. Given a possible position \( p \) of relay node, denote \( C(p) \) the set of sensor nodes, which can be covered by a relay node locating at position \( p \). Given a set \( H \) of possible positions, denote \( C(H) \) the set of sensor nodes, which can be covered by any relay node locating at a position in \( H \).

Algorithm:

For all \( s \in S \), \( H = \emptyset \); \( F = \emptyset \);
Pick \( q^* \in F \) such that \( q^* \) covers maximum number of sensor nodes in \( S \);
For all \( s \in S \cap C(q^*), \text{label}(s) = 1 \);
Step 1. For all \( q \in F \), find a pair of node-disjoint \( q^* - q \) paths, \( P_q \), in \( G^* \) with minimum sum of hop counts. For a fixed \( q \) and a sensor node \( s \), define \( \epsilon(s, P_q) \) as the number of times \( s \) is being covered by the relay nodes in \( P_q \cap F \), i.e. \( \epsilon(s, P_q) = \sum_{s \in C(q^* \cap F : s \in C(q^* \cap F)} \min\{\epsilon(s, P_q), 2 - \text{label}(s)\} \). Compute \( W_{P_q} = \sum_{s \in S} \min\{\epsilon(s, P_q), 2 - \text{label}(s)\} \).

Let \( q_0 \in F \) be the one with the highest ratio \( W_{P_{q_0}} / \sum_{s \in S} \min\{\epsilon(s, P_q), 2 - \text{label}(s)\} \).

For all \( s \in S \cap C(P_{q_0} \cap F), \text{label}(s) = \min\{\epsilon(s, P_q), 2 - \text{label}(s)\} \);
\( H = H \cup P_{q_0}, F = F - P_{q_0} \);
For all \( s \in S \), if \( \text{label}(s) = 2 \), remove \( s \) from \( S \);

Step 2. Construct graph \( G' \), by adding an artificial node \( v^* \) into \( G^* \), and connecting \( v^* \) with every node in \( H \). For all \( q \in F \), find the 2-node-disjoint \( v^* - q \) paths, \( P_q \), in \( G' \) with smallest sum of hop counts, compute \( W_{P_q} / \sum_{s \in S} \min\{\epsilon(s, P_q), 2 - \text{label}(s)\} \).

Let \( q_0 \in F \) be the one with the highest ratio \( W_{P_{q_0}} / \sum_{s \in S} \min\{\epsilon(s, P_q), 2 - \text{label}(s)\} \).

For all \( s \in S \cap C(P_{q_0} \cap F), \text{label}(s) = \min\{\epsilon(s, P_q), 2 - \text{label}(s)\} \);
\( H = H \cup P_{q_0}, F = F - P_{q_0} \);
For all \( s \in S \), if \( \text{label}(s) = 2 \), remove \( s \) from \( S \);

Step 3. If \( S = \emptyset \), stop, \( H \) is the solution; otherwise, repeat Step2.

It is not hard to see that our algorithm has a polynomial running time. Now, we prove the correctness of the algorithm.

Correctness of The Algorithm

In order to prove the correctness of the Algorithm, we need to show the following three things: (I) The algorithm will halt properly. (II) Given a solution obtained by the algorithm, every sensor node is covered by at least two relay nodes. (III) Given a solution, the relay nodes in the solution are 2-connected.

(I) If the graph \( G^* \) defined in the algorithm is 2-connected, then the algorithm will halt properly.

This is because that \( V(G^*) \) is a trivial solution for the problem, and if \( G^* \) is 2-connected, the process of the algorithm can keep going until either a solution of smaller size is found, or every node in \( V(G^*) \) has been chosen. We will show the 2-connectivity of \( G^* \) later in this subsection.

(II) Let \( H \) be the solution obtained by the algorithm. If we assume that the algorithm halt properly, then each sensor node is covered by at least two relay nodes in \( H \).

(III) Let \( H \) be the solution provided by the algorithm. The 2-connectivity of \( H \) will be the same as the 2-connectivity of the induced subgraph, \( G^*[H] \).

Hence, it suffices to show the 2-connectivity of \( G^*[H] \), which can be guaranteed by the algorithm as follows. Before Step1, there is only one node in \( H \); In Step1, we add in two node-disjoint paths, hence the current \( G^*[H] \) is 2-connected; When the algorithm loops through Step2, each time, by choosing the two node-disjoint paths from an unchosen node to the artificial node, the 2-connectivity is still preserved. Therefore, \( G^*[H] \) is 2-connected for the final solution \( H \).

Now, all we need to do is to show that \( G^* \) is 2-connected. Let \( u \) and \( v \) be two vertices of \( V(G^*) \). It suffices to show that there are two node-disjoint \( u-v \) paths in \( G^* \). This can be derived from the 2-connectivity of the sensor node set \( S \). Let \( s_u \) and \( s_v \) be two sensor nodes covered by \( u \) and \( v \), respectively. By the 2-connectivity of \( S \), there are two node-disjoint \( s_u-v \) paths, \( P^1_S \) and \( P^2_S \), in \( G_S \). Let \( P^1_S = s_u-s_1-s_2-\cdots-s_l-s_v \). \( P^1 \) be a possible position of relay node for \( s_u \) and \( s_1 \) (\( s_1 \) may be the same as \( u \)). Since we assume \( R \geq 2r \), \( distance(u, p_1) \leq R \), we have \((u, p_1) \in E(G^*) \). Let \( p_2 \) be a possible position of relay node for \( s_1 \) and \( s_2 \). By the same argument, we have \((p_1, p_2) \in E(G^*) \). Moving along the sensor nodes on \( P^2_S \) eventually, we can have a \( u-v \) path in \( G^* \). Similarly, we can have another \( u-v \) path, by moving along \( P^2_S \). Thus, we have shown that \( G^* \) is 2-connected. \( \square \)
We now analyze the performance of the algorithm.

**Definition.** (2, 1)-Distance: Given a graph \( G = (V, E) \), a 2-subset, \( T = \{x, y\} \), of \( V(G) \), and a node \( z \in V(G) \). Let \( P_1, P_2 \) be two node disjoint \( x-z, y-z \) paths, respectively, with the smallest sum of lengths. We call \( \text{length}(P_1) + \text{length}(P_2) \) the \( (2, 1) \) - Distance between \( T \) and \( z \), denoted \( (2, 1)\)-dist\((T, z)\).

\( (2, 1)\)-Diament: Given a graph \( G = (V, E) \). We define the \( (2, 1)\)-Diameter to be the maximum \( (2, 1)\)-Distance between a 2-node and a subset in \( V(G) \).

**Theorem 1:** Let \( R_{Alg} \) be the 2-Connected Relay Node Double Cover returned by our Algorithm. Let \( R_{OPT} \) be the optimal Relay Node Double Cover (not necessarily connected). We have

\[
\left| \frac{R_{Alg}}{R_{OPT}} \right| \leq D \cdot (1 + \log d),
\]

where \( D \) is the \((2, 1)\)-Diameter of \( G^* \), and \( d \) is the maximum number of sensor nodes which can be covered by one relay node.

**Proof.** Let \( S = \{s_1, \ldots, s_n\} \) be the set of sensor nodes. We make 2 copies of \( S \), i.e., \( S^* = S^1 \cup S^2 \), where \( S^j = \{s^j_1, \ldots, s^j_n\}, \forall j = 1, 2 \).

For each \( s^j_i \in S^* \), assign a weight \( w(s^j_i) = |P \cap F| / |WP| \), where \( P \) is the union of the two node-disjoint paths chosen during the execution of our Algorithm, such that before \( P \) is chosen label\([s_i]<j\), after \( P \) is chosen label\([s_i]=j\).

Let the optimal solution be \( R^* = \{q^*_1, \ldots, q^*_{|R_{OPT}|}\} \). Construct a bipartite graph \( B(S^2, R^*, E) \) as follows (Figure 1).

We build the edge set, \( E \), in \( |R_{OPT}| \) steps, starting from \( q^*_1 \), and moving toward \( q^*_{|R_{OPT}|} \). At each step \( i \), if \( q^*_i \) covers a sensor node \( s_j \), we connect \( q^*_i \) with \( s^h_j \), where \( h \) is the smallest integer s.t. \( s^h_j \) has not been connected to any \( q^*_i \) yet. Clearly, in \( B(S^2, R^*, E) \), each \( q^*_i \) has degree exactly one. For all \( q^*_i \in R^* \), let \( N_B(q^*_i) \) denote the set of neighbors of \( q^*_i \) in \( B(S^2, R^*, E) \). Let

\[
W(N_B(q^*_i)) = \sum_{s^y_j \in N_B(q^*_i)} w(s^y_j).
\]

Now, we find an upper bound of \( W(N_B(q^*_i)) \).

Suppose our algorithm loops for \( M \) iterations before it stops. At each iteration, we say a copy, \( s^j_i \), of sensor node \( s_h \) is covered, if label\([s_h]<j\); otherwise, call it uncovered. Let \( u_m \) be the number of uncovered elements of \( N_B(q^*_i) \) after iteration \( m \).

In particular, let \( u_0 = |N_B(q^*_i)| \).

Then, we have

\[
W(N_B(q^*_i)) = \sum_{m=1}^{M} (u_{m-1} - u_m) \frac{P_m \cap F}{P_m},
\]

where \( P_m \) is the union of the two node-disjoint paths chosen at iteration \( m \).

Obviously, \( |P_m \cap F| \leq D \), for any \( m \in \{1, \ldots, M\} \). Where \( D \) is the \((2, 1)\)-Diamater of \( G^* \). Due to the choice of \( P_1 \), we have \( W_{P_1} \geq u_0 \). Therefore

\[
\frac{W_{P_1}}{|P_1 \cap F|} \geq \frac{u_0}{D} \geq \frac{u_0 - u_1}{D}.
\]

Moreover, for all \( m \geq 2 \),

\[
\frac{W_{P_m}}{|P_m \cap F|} \geq \frac{W_{P_{m+1}^*}}{|P_{m+1}^* \cap F|} \geq \frac{u_m - u_{m+1}}{D},
\]

where \( P_{m+1}^* \) is the union of the two node-disjoint paths between \( H \) and \( q^*_i \) at iteration \( m \).

Now, we have

\[
W(N_B(q^*_i)) = \sum_{m=1}^{M} (u_{m-1} - u_m) \frac{|P_m \cap F|}{|P_m|} \leq D(1 + \sum_{m=2}^{M} \frac{u_m - u_{m-1}}{u_{m-1}}) \leq D(1 + \log u_0).
\]

Thus

\[
\sum_{q^*_i \in R^*} W(N_B(q^*_i)) \leq D(1 + \log u_0) \cdot |R^*|.
\]

Since each \( q^*_i \in S^2 \) has degree exactly one in \( B(S^2, R^*, E) \), we have

\[
\sum_{q^*_i \in R^*} W(N_B(q^*_i)) = \sum_{q^*_i \in R^*} \sum_{s^y_j \in N_B(q^*_i)} w(s^y_j) = \sum_{q^*_i \in S^*} \sum_{s^y_j \in N_B(q^*_i)} w(s^y_j) = \sum_{s^y_j \in S^*} w(s^y_j) = \sum_{s^y_j \in S} w(s^y_j) = |P_1|.
\]

Therefore, \( |R_{Alg}| \leq D \cdot (1 + \log u_0) \cdot |R_{OPT}| \). Since \( u_0 \) cannot exceed the maximum number \( d \) of sensor nodes which can be covered by one relay node, we have \( |R_{Alg}| \leq D \cdot (1 + \log d) \).

Be aware of the fact that the optimal solution of 2-Connected Relay Node Double Cover problem is also a solution for the Relay Node Double Cover problem (relay nodes are not necessarily to be connected). Hence, the performance ratio of our approximation solution against the optimal solution of Relay Node Double Cover problem is an upper bound of the performance ratio of the approximation solution. Since \( d \) is bounded by the number of sensor nodes, \( n \), we can claim that the solution provided by our algorithm is bounded within \( O(D \log n) \) times of the size of optimal solution.
V. Performance Evaluations

In this section, we evaluate the performance of our algorithm through simulation. We define the network density as \((n \cdot r^2) / f_s^2\), where \(n\) is the number of sensor nodes in the network, \(r\) is the communication range of the sensor node and \(f_s\) is the size of the playing field. In addition, we set the communication range of the relay node to 3 times of the sensor node’s communication range. We also formulate the RNDC problem by Integer Linear Programming (ILP) and use CPLEX to compute the optimal solution. This can be used as a lower bound on the optimal solution of our problem. It may be noticed that it is not even possible to give an ILP formulation for our problem, because it is impossible to identify finite possible positions for relay nodes.

Table I shows the error ratio of our algorithm with different network densities. The error ratio is defined as \(|R_{alg} - R_{opt}| / R_{opt}\). The \(R_{alg}\) is the size of 2CRNDC returned by our algorithm and \(R_{opt}\) is the size of the optimal RNDC. We control the network density by changing the number of sensor nodes. Moreover, we perform 10 trials for each specific network density and take the average. From the table, we can see that on average, error ratios of our algorithm are less than 25% under different network densities. What need to be mentioned is the size of optimal solution for RNDC problem is less than 2CRNDC problem, i.e. the ratio error ratio between our algorithm and the optimal solution for RNDC should be less. We compare our algorithm with optimal solution for RNDC problem since our performance analysis is based on that and we know that it supplies an upper bound for the performance ratio against optimal solution for 2CRNDC problem.

VI. Conclusions

In this paper, we have formulated a fault-tolerant relay node placement problem for wireless sensor networks. We have presented a polynomial time approximation algorithm to solve this problem and proved that its worst case performance ratio is bounded by \(O(D \log n)\). Preliminary simulation results shows that solutions provided by our algorithm are close to optimal. We are in the process of designing approximation algorithms with better performance ratios. Possible future research topics include extending our work to the more general \(k\)-connected relay node \(k\) cover problem.

REFERENCES


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