CIE 272
Mid-Term Examination #2
11/20/2009

1. All answers require the use of Table I in the text – cumulative probabilities for the standard normal distribution.

   Let AP and ap = April precipitation, and SEP and sep = September precipitation. To convert precipitation amounts into z-values we use the equations:

$$z = \frac{ap_o - \overline{ap}}{s_{AP}} = \frac{ap_o - 8.8}{3.5} \text{ and } z = \frac{sep_o - \overline{sep}}{s_{SEP}} = \frac{sep_o - 7.1}{2.8}$$

   a. P(SEP ≤ 5.0) = P(Z ≤ $\frac{5.0 - 7.1}{2.8}$) = P(Z ≤ -0.75). Find z = -0.75 in the Table:

      P(Z ≤ -0.75) = 0.226627. Therefore, **P(SEP ≤ 5.0 cm) = 0.226627**

   b. By definition, the 20th percentile has a cumulative probability of 0.20:

      P(AP ≤ ap_{25}) = 0.20

      Transforming AP to Z:

      P(AP ≤ ap_{25}) = P(Z ≤ $z = \frac{ap_{25} - 8.8}{3.5}$) = 0.20

      Find cumulative probability 0.20 in the Table. It is almost exactly at z = -0.84. Using this value:

$$z = -0.84 = \frac{ap_{25} - 8.8}{3.5}$$

      ***ap*₂₅ = -0.84(3.5) + 8.8 = 5.86 cm**

   c. Since April and September precipitation are independent,

      P(AP > 9.0 cm AND SEP < 6.0 cm) = P(AP > 9.0 cm) · P(SEP < 6.0 cm)

      P(AP > 9.0 cm):

      P(AP > 9.0 cm) = 1 – P(AP ≤ 9.0 cm)

$$z = \frac{9.0 - 8.8}{3.5} = 0.0571 \approx +0.06$$

      P(Z ≤ +0.06) = 0.524 = P(AP ≤ 9.0 cm)

      <u>P(AP > 9.0 cm) = 1 – 0.524 = 0.476</u>

      P(SEP ≤ 6.0 cm):

$$z = \frac{6.0 - 7.1}{2.8} = -0.393 \approx -0.39$$

      P(Z ≤ -0.39) = <u>0.348 = P(SEP ≤ 6.0 cm)</u>

Thus:

P(AP>9.0 cm AND SEP<6.0 cm) = P(AP > 9.0 cm) · P(SEP < 6.0 cm) = (0.476)·(0.348)

P(AP > 9.0 cm AND SEP < 6.0 cm) = 0.166

d. Since the precipitation is independent from year to year,

P(AP≤1.5 cm two consecutive years) = P(AP≤1.5 cm)·P(AP≤1.5 cm) = $[P(AP\leq 1.5 \text{ cm})]^2$

P(AP ≤ 1.5 cm):

$$z = \frac{1.5-8.8}{3.5} = -2.086 \approx -2.09$$

From the Table: P(Z ≤ -2.09) = 0.0183

P(AP≤1.5 cm two consecutive years) = $(0.0183)^2$ = 0.000342

This is a very low probability, so the wine maker can be fairly confident in the long-term viability of his business.

2. All answers require the use of Table I in the text – cumulative probabilities for the standard normal distribution.

a. Find z = +0.88 in the Table: **P(Z ≤ +0.88) = 0.810570**

b. Find cumulative probability 0.895 in the Table. It lies between **z = 1.25 and z = 1.26.** Any answer in this range is acceptable. The actual value is z = 1.2536

c. Interval probability: P(-1.81 ≤ Z ≤ -1.44) = P(Z ≤ -1.44) – P(Z ≤ -1.81). Find z = -1.44 and z = -1.81 in the Table: P(Z ≤ -1.44) = 0.074934; P(Z ≤ -1.81) = 0.035148. Therefore, P(-1.81 ≤ Z ≤ -1.44) = 0.074934 – 0.035148.

P(-1.81 ≤ Z ≤ -1.44) = 0.039786

d. If P(Z > z) = 0.210, then P(Z ≤ z) = 1 – 0.210 = 0.790.

Find the cumulative probability 0.790 in the Table. It lies between **z = 0.80 and z = 0.81.** Any answer in this range is acceptable. The actual value is z = 0.8064

e. Interval probability: P(+0.22 ≤ Z ≤ z) = P(Z ≤ z) – P(Z ≤ +0.22) = 0.375. Find z = +0.22 in the Table: P(Z ≤ +0.22) = 0.587064. Thus:

P(Z ≤ z) – 0.587064 = 0.375

So, P(Z ≤ z) = 0.375 + 0.587064 = 0.962064

Find the cumulative probability 0.962064 in the Table. It lies between **z = +1.77and z = +1.78.** Any answer in this range is acceptable. The actual value is z = +1.775.

3. In the answers below, the words "bad" and "good" refer to the condition of the weld. The words "pass" and "fail" refer to the results of a single inspection test, and "certified" and "rejected" refer to the conclusion made after the inspection procedure is completed according to the building code.

   a. **P(Bad weld Passes) = 1 – P(Bad weld Fails) = 1 – 0.90 = 0.10**

   b. The probability that *at least one weld fails* is hard to compute. However, it is easy if we realize that:

      P(at least one fails) = 1 - P(None Fail) = 1 – P(All Pass)

      The probability that 8 good welds all pass is just:

      P(Eight Good welds all Pass) = $[P(\text{Good weld Passes})]^8$

      P(Eight Good welds all Pass) = $(0.95)^8 = 0.66342$

      P(At least one weld fails) = 1 – 0.66342 = 0.33658

   c. A weld is only rejected if it fails two consecutive tests. Since these tests are independent:

      P(Good weld Fails Twice) = $[P(\text{Good weld Fails})]^2 = (0.05)^2$

      P(Good weld is Rejected) = 0.0025

      This is a very low probability, which is good.

   d. A bad weld could be certified if it passes the first inspection test (P = 0.10) **or** if it fails the first time (P = 0.90) **and** passes the re-test (P = 0.10):

      P(Bad weld Certified) = P(Bad weld Passes) + P(Bad weld Fails)·P(Bad weld Passes)

      P(Bad weld Certified) = (0.10) + (0.90)·(0.10) = 0.19

      This is too high of an "error rate" and the procedure should be modified to reduce this probability.