EXAM #3 – FALL 2008

1. a. $\left[DO\left(\frac{mg}{L}\right)\right] = -0.0123 \times \left[\text{Distance (km)}\right] + 0.3931$

If DO = 0.30 mg/L, then:

$0.30 = -0.0123 \times \text{Distance} + 0.3931$

$\text{Distance} = \frac{0.30 - 0.3931}{-0.0123} = \underline{\underline{7.57 \text{ km}}}$

b. Residual: $y_i - \hat{y}_i$ = measured – estimated

For $x = 12.8$ km:

$y_i = 0.25 \ \text{mg/L}$ (From Table of values)

$\hat{y}_i = -0.0123 \times (12.8) + 0.3931$

$= 0.236 \ \frac{mg}{L}$

$\text{Residual} = 0.25 - 0.236 = +\underline{\underline{0.014 \ \frac{mg}{L}}}$

c. $r = \frac{S_{xy}}{S_x \cdot S_y}$

$S_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{N-1} = \frac{-4.277}{8} = \underline{\underline{-0.5346}}$ (~~-0.0551~~)

$S_x = 6.60$, $S_y = 0.082$ } From Table

$\Rightarrow r = \frac{-0.5346 \ (\text{~~-0.0551~~})}{(6.60)(0.082)} = \underline{\underline{-0.988}}$

1. a. CI : $\bar{x} \pm t_{N-1;\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{N}}$

for recharge : $\bar{x} = 506$ $\quad S = 13.5$ $\quad N = 13$

For $\alpha = 0.05$: $t_{N-1;\frac{\alpha}{2}} = t_{12;0.025} = 2.179$

CI : $506 \pm 2.179 \cdot \frac{13.5}{\sqrt{13}} = \underline{506 \pm 8.2}$

$\underline{497.8 \text{ to } 514.2}$

b. Use either paired t-test or Satterthwaite Test

$H_1 : \mu_W - \mu_R > 0$ $\quad$ 'W' : withdrawal $\quad$ 'R' : recharge

$H_0 : \mu_W - \mu_R = 0$

Paired t-test : $\bar{D} = 32.5$ $\quad S_D = 25.3$ $\quad N = 13$ $\quad \delta = 0$

$$T = \frac{\bar{D} - \delta}{S_D/\sqrt{N}} = \frac{32.5 - 0}{25.3/\sqrt{13}} = 4.632$$

$t_{N-1;\alpha} = t_{12;0.05} = 1.782$

$T > t_{N-1;\alpha} \Rightarrow$ Reject H_0 $\quad$ Withdrawal Exceeds Recharge.

Satterthwaite Test :

$$T = \frac{(\bar{W} - \bar{R}) - \delta}{\sqrt{\frac{S_W^2}{N} + \frac{S_R^2}{N}}} = \frac{(538 - 506) - 0}{\sqrt{\frac{(25.9)^2}{13} + \frac{(13.5)^2}{13}}} = \underline{3.950}$$

$$f = d.f. = \frac{\left[\frac{(25.9)^2}{13} + \frac{(13.5)^2}{13}\right]^2}{\frac{\left(\frac{(25.9)^2}{13}\right)^2}{12} + \frac{\left(\frac{(13.5)^2}{13}\right)^2}{12}} = 18.1 \approx 18 \Rightarrow t_{f;\alpha} = t_{18;0.05} = \underline{1.734}$$

$T > t_{f;\alpha} \Rightarrow$ Reject H_0 $\quad$ Withdrawal Exceeds Recharge.

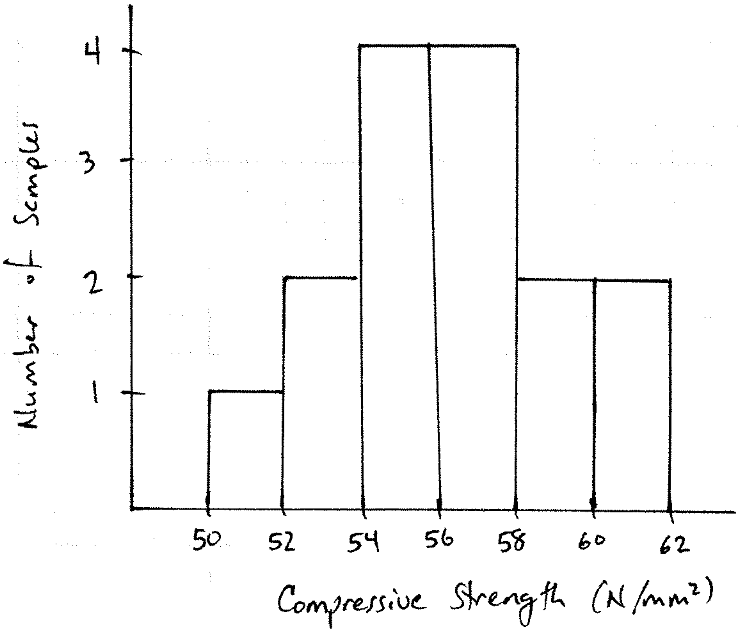
c. $H_1 : \mu_R > 495$ vs. $H_0 : \mu_R = 495$

$$T = \frac{\bar{R} - \mu_0}{s/\sqrt{N}} = \frac{506 - 495}{13.5/\sqrt{13}} = \underline{2.938} \qquad t_{N-1;\alpha} = t_{12;0.05} = \underline{1.782}$$

$T > t_{N-1;\alpha} \Rightarrow$ Reject H_0. $\quad$ Mean recharge is > 495

3\. a.

50-52 :	1
52-54 :	2
54-56 :	4
56-58 :	4
58-60 :	2
60-62 :	2



The histogram is a reasonable approximation of a 'bell curve' so we can say that these data are approximately normal.

b. C: $\bar{x} \pm t_{N-1;\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{N}}$ $\qquad \bar{x} = 56.3 \qquad s = 2.92 \qquad N = 15$

For $\alpha = 0.02$: $\quad t_{N-1;\frac{\alpha}{2}} = t_{14;0.01} = 2.624$

$$CI: \quad 56.3 \pm 2.624 \cdot \frac{2.92}{\sqrt{15}} = \underline{56.3 \pm 1.978}$$

OR $\underline{\underline{54.3 \text{ to } 58.3 \ N/mm^2}}$

c. $H_1 : \mu < 58$ vs. $H_0 : \mu = 58$

$$T = \frac{\bar{x} - \mu_0}{s/\sqrt{N}} = \frac{56.3 - 58}{2.92/\sqrt{N}} = \underline{-2.255}$$

$$t_{N-1;\alpha} = t_{14;0.05} = 1.761$$

$T < -t_{N-1;\alpha} \Rightarrow$ Reject H_0. The mean strength is less than 58 N/mm^2.