CIE 272
Mid-Term Examination #2
11/14/2008

1. All answers require the use of Table I in the text – cumulative probabilities for the standard normal distribution.

   a. Find $z = +1.83$ in the Table: **P(Z ≤ +1.83) = 0.966375**

   b. P(Z > -0.11) = 1 – P(Z ≤ -0.11). Find $z = -0.11$ in the Table: P(Z ≤ -0.11) = 0.456205. Therefore, **P(Z > -0.11) = 0.543795**

   c. Find cumulative probability 0.227 in the Table. It lies between **z = -0.75 and z = -0.74.** Any answer in this range is acceptable. The actual value is $z = -0.7488$.

   d. Interval probability: P(+0.16 ≤ Z ≤ +1.72) = P(Z ≤ +1.72) – P(Z ≤ +0.16). Find $z = 0.16$ and $z = 1.72$ in the Table: P(Z ≤ +1.72) = 0.957284; P(Z ≤ +0.16) = 0.563559. Therefore, P(+0.16 ≤ Z ≤ +1.72) = 0.957284 – 0.5635598.

      P(+0.16 ≤ Z ≤ +1.72) = 0.393725

   e. Interval probability: P(-1.16 ≤ Z ≤ z) = P(Z ≤ z) – P(Z ≤ -1.16) = 0.80. Find $z = -1.16$ in the Table: P(Z ≤ -1.16) = 0.123024. Thus:

      P(Z ≤ z) – 0.123024 = 0.80

      So, P(Z ≤ z) = 0.80 + 0.123024 = 0.923024

      Find the cumulative probability 0.923024 in the Table. It lies between **z = +1.42and z = +1.43.** Any answer in this range is acceptable. The actual value is $z = +1.426$.

2. All answers require the use of Table I in the text – cumulative probabilities for the standard normal distribution.

   Let T and t = asphalt thickness. Having set the paving machine to lay down 3.2 inches, we take 3.2 inches as the mean value. The standard deviation is given as 0.2 inches. To convert thicknesses into z-values we use the equation:

$$z = \frac{t - \bar{t}}{s_t} = \frac{t - 3.2}{0.2}$$

   a. $P(T \leq 3.0) = P(Z \leq \frac{3.0 - 3.2}{0.2}) = P(Z \leq -1.0)$. Find $z = -1.0$ in the Table:

      P(Z ≤ -1.0) = 0.158655. Therefore, **P(T ≤ 3.0 inches) = 0.158655**

b. Interval probability: $P(3.0 \le T \le 3.5) = P(\frac{3.0-3.2}{0.2} \le Z \le \frac{3.5-3.2}{0.2})$

Thus, $P(3.0 \le T \le 3.5) = P(-1.0 \le Z \le +1.5) = P(Z \le +1.5) - P(Z \le -1.0)$

Find $z = +1.50$ and $z = -1.0$ in the Table:

$P(Z \le +1.5) = 0.933193$; $P(Z \le -1.0) = 0.158655$.

$P(Z \le +1.5) - P(Z \le -1.0) = 0.933193 - 0.158655 = 0.774538$

Thus, $P(3.0 \le T \le 3.5) = 0.774538$, and we can say that **77.5% of the asphalt on the road should have thickness between 3.0 and 3.5 inches.**

c. By definition, the 75th percentile has a cumulative probability of 0.75:

$P(T \le t_{75}) = 0.75$

Transforming T to Z:

$$P(T \le t_{75}) = P(Z \le z = \frac{t_{75} - 3.2}{0.2}) = 0.75$$

Find cumulative probability 0.75 in the Table. It lies between $z = +0.67$ and $z = +0.68$. Using either value for this exam is ok. I used $z = +0.675$.

$$z = 0.675 = \frac{t_{75} - 3.2}{0.2}$$

$\boldsymbol{t_{75} = 0.675(0.2) + 3.2 = 3.335}$ **inches**

d. $$P(T > 3.0''|T > 2.8'') = \frac{P(T > 3.0'' \text{ AND } T > 2.8'')}{P(T > 2.8'')}$$

Note that if the thickness is greater than 3.0 inches, it MUST be greater than 2.8 inches too! Therefore:

P(T > 3.0” AND T > 2.8”) is just equal to P(T > 3.0”)!

$$P(T > 3.0''|T > 2.8'') = \frac{P(T > 3.0'')}{P(T > 2.8'')} = \frac{1 - P(T \le 3.0'')}{1 - P(T \le 2.8'')}$$

$P(T \le 3.0'') = 0.158655$ (from part a.)

$P(T \le 2.8'') = P(Z \le \frac{2.8-3.2}{0.2}) = P(Z \le -2.0)$. Find $z = -2.0$ in the Table:

$P(Z \le -2.0) = 0.022750$. Therefore, $P(T \le 2.8 \text{ inches}) = 0.022750$

$$P(T > 3.0''|T > 2.8'') = \frac{1 - 0.158655}{1 - 0.02275} = 0.861$$

P(T > 3.0 inches | T > 2.8 inches) = 0.861

3. Remember that for independent events A and B, P(A and B) = P(A)P(B). In the answers below, "Found" is the Foundation phase and "Super" is the Superstructure phase.

   a. P(Found = 5 mo. **and** Super = 6 mo.) = P(Found = 5 mo.)P(Super = 6 mo.)

      Look up the probabilities for the *standard* speed:

      P(Found = 5 mo.) = 0.70 for *standard* speed.

      P(Super = 6 mo.) = 0.40 for *standard* speed.

      P(Found = 5 mo. and Super = 6 mo.) = (0.7)(0.4) = 0.28

   b. The foundation may take either 4 or 5 months. The superstructure may take either 5, 6, or 7 months. The only ways for the entire structure to be built in exactly 10 months are:

      i. Found = 4 mo. and Super = 6 mo.

      ii. Found = 5 mo. and Super = 5 mo.

      Since these are disjoint (mutually exclusive), we can write:

      P(Exactly 10 months) = P(Found = 4 and Super = 6) + P(Found = 5 and Super = 5)

      And, since the foundation and superstructure phases are independent:

      P(Exactly 10 months) = P(Found = 4)P(Super = 6) + P(Found = 5)P(Super = 5)

      Look up the foundation probabilities for the *fast* speed:

      P(Found = 4 mo.) = 0.50 for *fast* speed.

      P(Found = 5 mo.) = 0.50 for *fast* speed.

      Look up the superstructure probabilities for the *standard* speed:

      P(Super = 5 mo.) = 0.10 for *standard* speed.

      P(Super = 6 mo.) = 0.40 for *standard* speed.

      P(Exactly 10 months) = (0.5)(0.4) + (0.5)(0.1) = 0.25

c. This problem can be solved by adding up all of the probabilities that will lead to the building being constructed in 9, 10, or 11 months. However, it is much easier to solve this problem by using the *complement*:

P(Building ≤ 11 months) = 1 – P(12 months)

Note that 12 months is the longest it could possibly take, and that the **only** combination that combination that would result in 12 months is 5 months for the foundation and 7 months for the superstructure.

P(12 months) = P(Found = 5 mo.)P(Super = 7 mo.)

Look up the probabilities for the *fast* speed:

P(Found = 5 mo.) = 0.50 for *fast* speed.

P(Super = 7 mo.) = 0.35 for *fast* speed.

Thus, P(12 months) = (0.5)(0.35) = 0.175

And, **P(Building ≤ 11 months) = 1 – P(12 months) = 0.825**

d. Using the same approach as part c, find all of the combinations (Foundation/Superstructure) that yield P(12 months) ≤ 0.15:

- Express/Express: P(12 mo.) = 0.06 Cost: $535k
- Express/Fast: P(12 mo.) = 0.105 Cost: $485k
- **Express/Standard: P(12 mo.) = 0.15 Cost: $420k**
- Fast/Standard: P(12 mo.) = 0.10 Cost: $502k
- Standard/Express: P(12 mo.) = 0.14 Cost: $481k

The least expensive alternative that meets the probability criterion is **Express construction for the foundation and Standard construction for the superstructure.**