CIE 272

Mid-Term Exam #3

December 12, 2003

1. (a) $r = \dfrac{s_{xy}}{s_x s_y}$

   s_x = 3,769 s_y = 6,418

   $$s_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y}) = \frac{1}{12}(265{,}973{,}745) = 22{,}164{,}479$$

   $$r = \frac{22{,}164{,}479}{3769 \cdot 6418} = 0.916$$ *This is a **strong** correlation.*

   (b) Use bar diameter because the relationship with the elastic limit is more linear.

2. (a) The paired t-test **is** appropriate because the same intersections are studied before and after the traffic lights were installed.

   (b) $F = \dfrac{(7.48)^2}{(5.47)^2} = 1.87$

   $k_1 = k_2 = 12$

   $F_{a/2;k_1-1;k_2-1} = F_{0.025;11;11} = 3.53$

   Since $F < F_{crit}$, we accept the null hypothesis that the **variances are equal**.

   (c) $S_p^2 = \dfrac{11(7.48)^2 + 11(5.47)^2}{22} = 42.94$

   S_p = 6.55

   $$T = \frac{15.7 - 10.2}{6.55\sqrt{\frac{1}{12} + \frac{1}{12}}} = 2.06$$

   $T_{Nx+Ny-2;a} = T_{22;0.01} = 2.508$

   Since $T < T_{crit}$, we accept the null hypothesis; there is no evidence that the installation of the street lights has resulted in fewer accidents. [However, it would be better to use the paired t-test.]

3. (a) $T = \dfrac{\bar{x} - 850}{s/\sqrt{N}}$

   (b) Reject H_o if $T > t_{20;0.05} = 1.725$ (this is a one-tailed test).

(c) $T = \dfrac{\bar{x} - 850}{300/\sqrt{N}}$ This value will be compared to $t_{N-1;0.05}$. The result will be significant when T is infinitesimally greater than $t_{N-1;0.05}$. From part (b), I know that if N = 21, the t value is 1.725, so let's start there:

$1.725 = \sqrt{N}\dfrac{1000-850}{300} = 0.5\sqrt{N}$ Thus, N = 12. Now use N = 12:

$t_{N-1;0.05} = t_{11;0.05} = 1.796 = \sqrt{N}\dfrac{1000-850}{300} = 0.5\sqrt{N}$ Now, N = 13:

$t_{N-1;0.05} = t_{12;0.05} = 1.782 = \sqrt{N}\dfrac{1000-850}{300} = 0.5\sqrt{N}$ Now, N = 13 (No change)

Therefore, **13 samples would be required.**

4. (a) *slope* = A; *intercept* = C_∞

Using the equation:

$$A = 203.6$$

$$C_\infty = 1.7$$

(b) The units of C must be mg/kg. So A/d and C_∞ must also have those units. Since d is in cm:

$$A : \frac{mg \cdot cm}{kg} \qquad C_\infty : \frac{mg}{kg}$$

(c) $\displaystyle\int_{d=2.5cm}^{d=150cm}\left(\frac{A}{d} + C_\infty\right)dd = \left(A \cdot \ln(d) + C_\infty \cdot d\right)\Big|_{2.5}^{150}$

$= [203.6 \cdot \ln(150) + 1.7 \cdot (150)] - [203.6 \cdot \ln(2.5) + 1.7(2.5)] = 1084.4$

Mass of PCB's = 0.015×1084.4 = **16.3 g**