CIE 272

Mid-Term Exam #2

November 17, 2003

1. (a) **$P(Z \leq -1.77) = 0.03836$**. read directly from standard normal table.

   (b) Find 0.015 in the standard normal table; it is very close to **$z = -2.17$**

   (c) $P(0.51 \leq Z \leq 2.00) = P(Z \leq 2.00) - P(Z \leq 0.51)$

   Use the standard normal table to find these cumulative probabilities:

   $P(Z \leq 2.00) = 0.97225$

   $P(Z \leq 0.51) = 0.69497$

   Therefore, **$P(0.51 \leq Z \leq 2.00)$ =** $0.97225 - 0.69497 =$ **0.28228**

   (d) If $P(Z > z) = 0.039$, then $P(Z \leq z) = 0.961$.

   Use the standard normal table to find the z-value that yields a cumulative probability nearest to 0.961:

   $P(Z \leq +1.76) = 0.960796$

   Thus, **$z = +1.76$**

   (e) $P(-1.18 \leq Z \leq z) = 0.60 = P(Z \leq z) - P(Z \leq -1.18)$

   First, look up the probability $P(Z \leq -1.18)$ in the standard normal table. It is 0.119.

   Now, using that value:

   $0.60 = P(Z \leq z) - 0.119$

   So, $P(Z \leq z) = 0.60 + 0.119 = 0.719$

   Look up the z-value corresponding to a probability of 0.719 in the table. It lies very close to **$z = +0.58$**.

2. (a) The ranks of the median and quartile values are:

   $$\text{Median Rank} = \frac{N+1}{2} = \frac{47}{2} = 23.5$$

   $$\text{Lower Quartile Rank} = \frac{N+3}{4} = \frac{49}{4} = 12.25$$

   $$\text{Upper Rank} = \frac{3N+1}{4} = \frac{139}{4} = 34.75$$

   Therefore the median is the average of the 23^{rd} and 24^{th} values, which are 6.3 and 6.4:

   Median = 6.35

The lower quartile is the average of the 12th and 13th values, which are 6.0 and 6.1:

Lower Quartile = 6.05

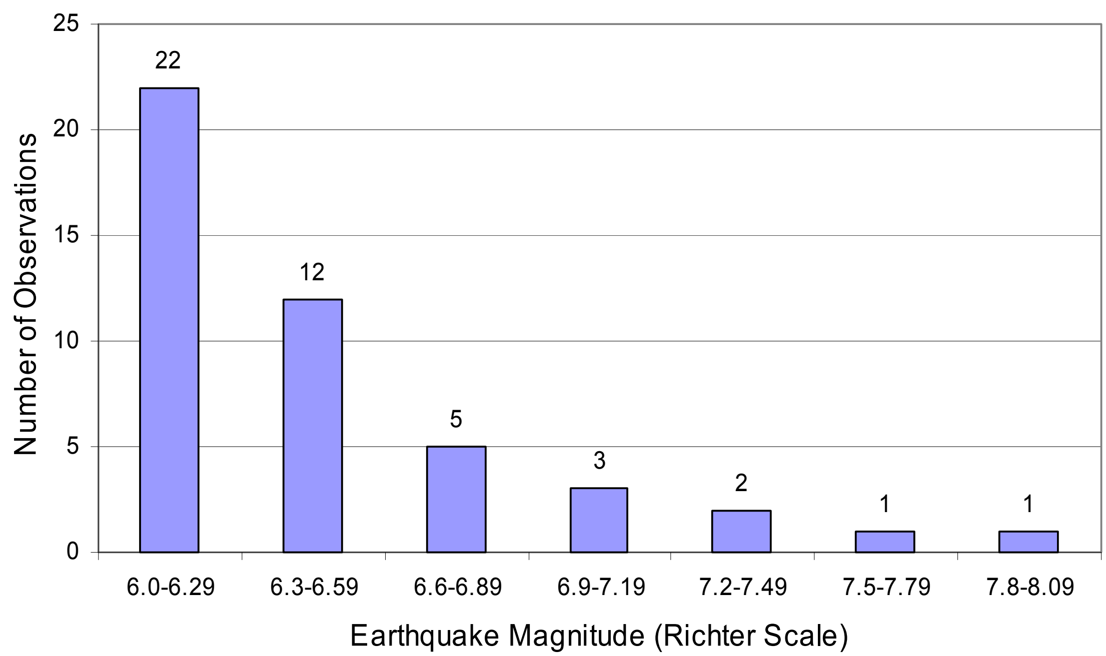
The upper quartile is the average of the 34th and 35th values, which are 6.5 and 6.6:

Upper Quartile = 6.55

(b) Count the number of times each value was observed:

Interval	Number of Observations
6.0 – 6.29	22
6.3 – 6.59	12
6.6 – 6.89	5
6.9 – 7.19	3
7.2 – 7.49	2
7.5 – 7.79	1
7.8 – 8.09	1

The histogram is a bar chart of these values:



(c) **This histogram looks nothing like a bell-shaped curve, so it is not reasonable to assume that the earthquake magnitude is normally distributed.**

3. The basic probabilities in this problem are:

P(A fails) = 0.02 therefore: P(A works) = 0.98

P(B fails) = 0.02 therefore: P(B works) = 0.98

P(C fails) = 0.15 therefore: P(C works) = 0.85

(a) **P(A works) = 0.98** on any day it is in service.

(b) P(A fails *and* B fails) = P(A fails)P(B fails) = (0.02)(0.02) = **0.0004**

(c) P(at least one pump fails on Saturday) = 1 – P(no pump fails on Saturday)

= 1 – P(A works)P(C works)

= 1 – (0.98)(0.85)

P(at least one pump fails on Saturday) = 0.167

(d) P(B works *and* C fails) = P(B works)P(C fails) = (0.98)(0.15)

P(B works *and* C fails) = 0.147

(e) P(no failures in a week). This can be approached in several ways. Here's one way to look at it: if there are no failures, then pump A must work on six days, pump B must work on six days, and pump C must work on two days. Since the pumps and days are all independent:

P(no failures in a week) = $(0.98)^6(0.98)^6(0.85)^2$ = 0.567

4. (a) The 99% confidence interval is given by:

$$\bar{x} \pm t_{\alpha/2;N-1} \cdot \frac{s}{\sqrt{N}}$$

For the fraction of failed piles:

$\bar{x} = 0.051$

$s = 0.026$

$N = 10$ (number of samples)

For a 99% confidence level, $\alpha = 0.01$, so $\alpha/2 = 0.005$. Thus, the value of the t-statistic is $t_{0.005;9}$. Using the t-table, the corresponding value is 3.250. Therefore, the confidence interval for the fraction of failed pile is computed as:

$$0.051 \pm 3.250 \cdot \frac{0.026}{\sqrt{10}} = 0.051 \pm 0.027$$

or: **0.024 to 0.078**

(b) The safe approach is to use the upper CI value to estimate the fraction of failed piles. If we do this, then we can be more than 99% sure that at least 75 of the piles will be successful.

If the contractor drives N piles, and the fraction that fails is 0.078, then:

$$N_{\text{successful}} = 75 = (1 - 0.078)N$$

Solving for N:

$$N = \frac{75}{(1 - 0.078)} = 81.3$$

You can't drive a fraction of a pile, so **the contractor must be prepared to drive 82 piles.**