CIE 272

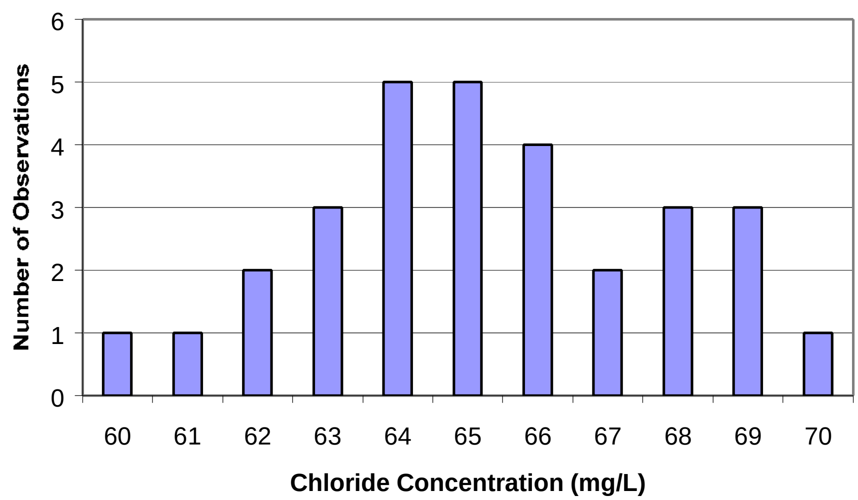
Mid-Term Exam #2

11/11/02

1. (a) Count the number of times each value was observed:

Chloride Concentration (mg/L)	Number of Observations
60	1
61	1
62	2
63	3
64	5
65	5
66	4
67	2
68	3
69	3
70	1

The histogram is a bar chart of these values:



(b) The form of the histogram is reasonably close to a bell-shaped curve, so it is reasonable to assume that the chloride concentration is normally distributed.

(c) Standard Error $= \frac{s}{\sqrt{N}} = \frac{0.200}{\sqrt{30}} = 0.0365$ mg/L

Coefficient of Variation = $\text{cv} = \frac{s}{\bar{x}} \cdot 100 = \frac{0.200}{1.82} \cdot 100 = 10.99\%$

(d) The 99% confidence interval is given by:

$$\bar{x} \pm t_{\alpha/2;N-1} \cdot \frac{s}{\sqrt{N}}$$

For phosphate:

$\bar{x} = 1.82$ mg/L

s = 0.200 mg/L

N = 30 (number of samples)

For a 99% confidence level, $\alpha = 0.01$, so $\alpha/2 = 0.005$. Thus, the value of the t-statistic is $t_{0.005;29}$. Using the t-table, the corresponding value is 2.756.

Therefore, the confidence interval for the phosphate concentrations is computed as:

$$1.82 \pm 2.756 \cdot \frac{0.200}{\sqrt{30}} = 1.82 \pm 0.10$$

or

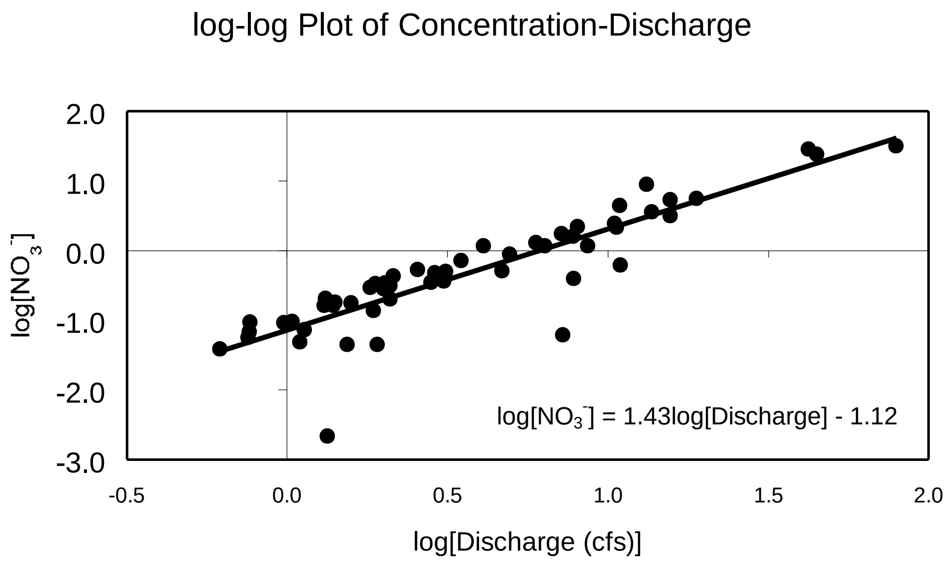
1.72 to 1.92

(e) Use the frequency to estimate the probability. Of the 13 days when the chloride concentration was greater than 65 mg/L, the phosphate concentration was greater than 2 mg/L on 3 of them. Thus:

Prob(Phosphate > 2 mg/L | Chloride > 65 mg/L) ~ 3/13, or 0.231

(f) If the events are independent, then the conditional probability in (e) should equal the unconditional probability that the phosphate concentration is greater than 2 mg/L. This probability can be estimated as 4/30, or 0.133. This is a good deal lower than the value computed in (e), so I would conclude that the occurrences are not independent.

2. (a) The regression line is shown in the graph below. Other reasonable answers are acceptable.



(b) For a log-log plot, the general equation relating *x* and *y* is:

$$y = 10^{intercept} \cdot x^{slope}$$

From the equation for the line, we see that the intercept is –1.12, and the slope is +1.43, so:

$$[NO_3^-] = 0.0759(\text{Discharge})^{1.43}$$

(c) For Discharge = 65 $ft^3\ sec^{-1}$:

$$[NO_3^-] = 0.0759\ (65)^{1.43}$$

$$[NO_3^-] = 29.7\ \text{mg/L}$$

Inspecting the original graph, this value makes sense.

3. (a) $P(0.58 = Z \leq 0.82) = P(Z \leq 0.82) – P(Z \leq 0.58)$

Use the standard normal table to find these cumulative probabilities:

$P(Z \leq 0.82) = 0.793892$

$P(Z \leq 0.58) = 0.719043$

Therefore, $P(0.58 = Z \leq 0.82) = 0.793892 – 0.719043 = 0.07485$

(b) $P(Z > 1.59) = 1 – P(Z \leq 1.59)$

Use the standard normal table to find:

$P(Z \leq 1.59) = 0.944083$

Thus, $P(Z > 1.59) = 1 – 0.944083 = 0.055917$

(c) If $P(Z > z) = 0.588$, then $P(Z \leq z) = 1 – 0.588 = 0.412$.

Look up 0.412 in the standard normal table. It lies between z = -0.22 and z = -0.23.

z = -0.222

(d) $P(z \leq Z \leq 1.26) = 0.75 = P(Z \leq 1.26) – P(Z \leq z)$

First, look up the probability $P(Z \leq 1.26)$ in the standard normal table. It is 0.896.

Now, using that value:

$0.75 = 0.896 – P(Z \leq z)$

So, $P(Z \leq z) = 0.896 – 0.75 = 0.146$

Look up the z-value corresponding to a probability of 0.146 in the table. It lies between z = –1.05 and z = –1.06.

z = -1.053