Two rakes of cross-wire probes were used to capture the two-point velocity statistics in a flow through an axisymmetric sudden expansion. The expansion ratio of the facility is 3, and has a constant geometry. Measurements were acquired at a Reynolds number equal to 54,000 based on centerline velocity and inlet pipe diameter. The two-point velocity correlations were obtained along a plane normal to the flow \((r, \theta)\), at eleven downstream step-height positions spanning from the recirculating region, through reattachment, and into the redeveloping region of the flow. Measurements were acquired by means of a flying hot-wire technique to overcome rectification errors near the outer wall of the pipe where flow recirculations were greatest. A mixed application of Proper Orthogonal (in radius) and Fourier Decomposition (in azimuth) was performed at each streamwise location to

† Laboratoire d’Etudes Aérodynamiques - UMR CNRS 6609, Université de Poitiers, France
‡ Present address: Ford Motor Company, Dearborn, MI, NY, USA
provide insight into the dynamics of the most energetic modes in all regions of the flow. Information from this multi-point analysis reveals that the flow evolves from the Fourier-azimuthal mode $m = 2$ (containing the largest amount of turbulent kinetic energy) in the recirculating region, to $m = 1$ in the reattachment and redeveloping regions of the flow. The results manifest a plausible feedback model for the recirculating region of the flow to explain its overall low-order behavior, when compared to other similar flows such as the axisymmetric jet. The driving mechanisms for the unique feedback structure (an axisymmetric mode), are the higher Fourier-azimuthal modes $m = 4$ and $5$, that are shown to follow the streamline curvature precisely to reattachment, where the feedback cycle is initiated by the genesis of the axisymmetric mode. The lack of any organized structures along the outer wall after reattachment infers a collision of the structures with the outer wall, rather than an alternate convection up and downstream at reattachment.

1. Introduction

Understanding the dynamic behavior of separated flows is of immense importance for improving and manipulating many practical fluid engineering problems. The axisymmetric sudden expansion is a key canonical separated flow that exhibits this highly complex flow phenomenon. Industrial applications of sudden expansions are seen in the automotive and HVAC industry. In addition, sudden expansions are examined extensively in the propulsion industry because of their similarities to sudden dump combustors. Swirl is introduced at the expansion, and the recirculation region is used as a flame holder. Several reasons for studying this combustion process include, (1) improved flame stability, (2) providing for more complete mixing of atomized fuel agents, (3) controlling inlet temperatures to the turbine, and (4) reducing emissions of oxides of nitrogen and un-
burned hydrocarbons relevant to ozone depletion. From a scientific standpoint, insightful information about the fundamental behaviors of massively separated flows can be learned using sudden expansion type geometries.

The flow through an axisymmetric sudden expansion is three dimensional in nature. Because of the flow’s mean azimuthal invariance, much attention has been given to the highly inhomogeneous nature of the radial and streamwise plane. It is in this plane that there exist three regions of interest; recirculating, reattaching, and redeveloping, (figure 1). As the flow passes over the expansion lip, a free shear layer is formed thus creating a high-speed side (located near the free-stream flow) and a low-speed side, (located near the wall). Eventually the outer shear layer migrates towards the wall, thus impacting its surface and establishing a reattachment region. Roth & Johnston (1976) defined the reattachment region as the physical location at which the instantaneous velocity was in either forward or reverse flow 50% of the time, and where the Reynolds stresses were found to decay most rapidly.

Early investigations of axisymmetric expansions by Chaturvedi (1963) used pitot tubes and CTA techniques (single-wire probes) to obtain mean and fluctuating profiles,
respectively. These investigations found a noticeable mutation in the turbulent structures, outside of the recirculating region near reattachment, from their original state. Unfortunately, the findings were limited by the unavailability of suitable measuring techniques for high turbulence intensity reversing flows and the quality of their results are enigmatic. Bradshaw & Wong (1972) later investigated the simple back-step flow and discovered that the Reynolds stress decay was brought upon by the presence of a strong adverse pressure gradient and the absence of a normal velocity ($v = 0$) at the wall, causing large eddies to be torn in two. Opposing arguments on flow reattachment from Kim et al. (1978) concluded that some eddies were swept upstream and some downstream. Furthermore, Eaton & Johnston (1981) identified five parameters that governed the reattachment length: (1) initial boundary layer state, (2) initial boundary layer thickness, (3) free stream turbulence, (4) pressure gradient, and (5) aspect ratio. The reader is referred to Eaton & Johnston (1981) for a more comprehensive discussion of results using hot-wire, pulsed-wire, laser anemometry and flow visualization techniques.

Recently, Cole & Glauser (1998b) illustrated the flow’s unsteady behavior through the axisymmetric sudden expansion using conditional eddy estimation techniques, via multiple channels of simultaneously sampled CTA probes. They reconstructed an instantaneous estimate of the turbulent structure in the $r, z$ plane based on a prior knowledge about the statistical relationship between the core and recirculating regions of the flow, (this technique utilized the flying-wire system used in the present study and is described in §3.1). This involved tactfully placing stationary cross-wire probes in the flow’s core region where turbulence intensities were moderate and there was no reverse flow. In doing so, they avoided rectification errors ordinarily experienced with CTA tools when trying to measure high turbulence intensity flows, and were able to properly estimate the flow’s highly unsteady near-wall region. It had been concluded from these mea-
measurements, that coherent structures from the near-field shear layer (upstream near the expansion lip) had propagated towards the wall, thus causing the unsteadiness in the reattachment region. These findings were similar to Johnston (1976), who reportedly observed large scale structures on the order of the step-height convecting through this region. The investigation of Johnston (1976) had concentrated on the $r, z$ plane of the flow over a backward-facing step, and incorporated a variable geometry with half angles of 15°, 30°, 45°, & 90°, using LDA measurement tools.

Additional findings from Cole & Glauser (1998a) concluded that for an expansion ratio of 3, the turbulent kinetic energy ($TKE$) was found highest between five and eight step-heights downstream from the expansion lip, at $r/R \approx 0.2$. This was thought to occur near the core because of the merging of the inner shear layers. Also, the $TKE$ production was observed to reach its maximum within the first four step-heights. These were consistent findings with those of Stokes (1999) and Hussein et al. (1994) in the backward-facing step and the axisymmetric jet's shear layer, respectively. Cole & Glauser (1998a) also found that the reattachment length was about 9.0 step-heights from the expansion lip.

These previous investigators focused on the $r, z$ plane of the axisymmetric sudden expansion. Thus, there is a lack of clarity regarding the role of the azimuthal structure on the mean turbulent motions of these flows, especially in the reattachment region where the flow has been shown to be most unsteady. This paper presents an investigation where we employ multi-point flying hot-wire measurements with low dimensional analytical tools to elucidate the physics of the flow in the $r, \theta$ plane of the axisymmetric sudden expansion. This is executed at several streamwise locations, covering all three regions of the flow; recirculating, reattaching and redeveloping.
1.1. Identifying Turbulent Structures

The importance of large scale structures in turbulent flows has been known for quite some time. The dynamics of these structures have been shown to control many of the events that characterize turbulence, and has been a topic of some debate. Qualitative models of coherent structures by Townsend (1956), showed finite correlations that accounted for 10% to 20% of the flow’s total energy. These structures were initially believed to play a role in the flow’s entrainment and intermittency, and considered only as passive contributors to the dynamics of turbulence. Brown & Roshko (1974) used a novel apparatus to clearly show the existence of coherent structures, and their evolution in a plane turbulent mixing layer. They concluded that the large scale interactions governed the growth of the shear layer, and that their energy was transformed through fluidic amalgamation, a process of vortex pairing and rotation that was described by Winant & Browand (1974).

Many visualization studies have clearly shown that coherent structures play a key role in the dynamics of turbulent shear flows, but such results are qualitative and limited. The recent development and availability of more sophisticated tools have provided evidence that these large energetic structures contain a moderate percentage of the energy in shear flows, shifting the focus towards examining the evolution of these structures in all three dimensions.

Lumley (1967) suggested a quantitative technique for identifying the more energetic structures in turbulent flows. The analysis requires that an optimization of a random field be represented by a set of deterministic functions which are in turn functions of the field itself. The early work of Glauser & George (1987a) utilized this Proper Orthogonal Decomposition (POD) technique in the turbulent shear layer of the incompressible axisymmetric jet and showed that the dominant basis function (1st POD mode) contained 40% of the total $TKE$, with an additional 40% of the energy found in the next two modes.
An extension of this study by Citriniti & George (2000) and Gamard et al. (2002) captured the temporal evolution of the jet’s $r, \theta$ plane using a highly dense grid of 138 single-wire probes (Reynolds number of $8E^5$). This later investigation used the POD to filter out small scale fluctuations, capable of obscuring the large scale interactions in the turbulent shear layer. Ukeiley et al. (1999) and Taylor et al. (2001) performed similar studies of the $r, \theta$ plane in the turbulent sound-source regions of the Mach 0.3 & 0.6 compressible axisymmetric jet. They concluded that the dominant POD mode converged rapidly and contained over 40% of the fluctuating mean square mass flux.

Other successful applications of this technique to transitional and turbulent flows have been performed, [e.g. Moin & Moser (1989) Glauser & George (1987b), Citriniti & George (2000), Rempfer (2003), Taylor et al. (2001), Gamard et al. (2002), Delville et al. (1999), Ukeiley et al. (2001), Gordeyev & Thomas (2000) and Glauser et al. (2004)]. For more detail of the POD’s mathematical construction, its historical applications to turbulent flows, as well as its relation to other modelling techniques, i.e. Galerkin projection, Linear Stochastic Estimation, pattern recognition, and others, the reader is referred to Berkooz et al. (1993) and Glauser & George (1992). The use of these low-dimensional techniques, have added a wealth of knowledge to understanding the interactive nature of the more energetic turbulent features in shear flows. The axisymmetric sudden expansion is a natural extension to the aforementioned jet studies because it is also a canonical flow, but with the additional complexity of being massively separated and wall-bounded.

In the present study, Proper Orthogonal and Fourier Decomposition techniques are applied to the streamwise and radial fluctuating velocities in the flow’s $r, \theta$ plane of the axisymmetric sudden expansion. This is performed independently at several step-height locations ($z/h = 3$ to 13) downstream from the expansion lip ($z/h = 0$). This
work complements the work of Cole (1996) who applied the 1-d POD, in radius, to the fluctuating velocity field at several streamwise locations in the same facility. Cole (1996) concluded that the first POD mode consistently captured between 35% and 40% of the TKE, at $z/h = 2$ through 13, and nearly 50% of the energy with the addition of the second POD mode.

The motivation of this study stems from an interest in understanding the importance of the azimuthal structure in flows with wall-bounded axisymmetric geometries. The low dimensional techniques that are employed are discussed in § 2, followed by a description of the experiment, the flying-wire technique, and a discussion of the basic statistical features of the flow through the axisymmetric sudden expansion in § 3. Radial and azimuthal correlations are presented and discussed in § 4 in a manner fitting to the analytical tools employed. The results of the decomposition are shown in § 5 including a modal reconstruction of the kernel and a model describing the feedback mechanism, to demonstrate the dominant characteristics of this wall bounded flow.

2. Proper Orthogonal Decomposition

The Proper Orthogonal Decomposition has been shown to be an effective technique for decomposing a random field of vectors into a characteristic basis set, from which the maximization of the ordered basis defines the most probable vector from that set, in the mean square sense. This maximization relies on selecting a candidate event with the largest mean square projection on the vector field $\vec{u}$ (via the calculus of variations), and the normalization follows from $\langle | |^2 \rangle = \langle | \vec{u}, \phi \rangle^2 \rangle / | \phi, \phi^\ast |$, where $<>$ denotes ensemble averaging. The kernel used in the maximization is constructed using Hilbert-Schmidt’s theory of integral equations with symmetric kernels, and the problem reduces to solving the integral equation (2.1) for an infinite number of eigenvalues. Some of the more general
mathematical properties are outlined in Aubry et al. (1988) and Berkooz et al. (1993).

\[ \int \int R_{ij}(\vec{x}, \vec{x}') \phi_j(\vec{x}') d\vec{x} = \lambda \phi_i(\vec{x}) \]  

Figure 2. Cylindrical coordinate system in the axisymmetric sudden expansion.

Here the kernel \( R_{ij} \) is the velocity (ensemble averaged) two-point cross-correlation tensor created from a series of discrete stationary turbulent flow events, \( R_{ij}(\vec{x}, \vec{x}') = \langle u_i(\vec{x}, t) u_j(\vec{x}', t) \rangle \), and the eigenvalue \( \lambda \) is equal to \( \langle |\alpha|^2 \rangle \). The technique is optimal in that most of the energy is contained in the first structure alone, and the orthonormal sequence is ordered, \( (\lambda^{(n)} \geq \lambda^{(n+1)}) \). Since the eigenvalues and eigenfunctions are properties of the kernel, they can be utilized to reconstruct it following (2.2).

\[ R_{ij}(\vec{x}, \vec{x}') = \sum_{n=1}^{\infty} \lambda^{(n)} \phi_i^{(n)}(\vec{x}) \phi_j^{(n)}(\vec{x}') \]  

An infinite number of eigenfunctions can be used to reconstruct the original instantaneous velocity:

\[ u_i(\vec{x}, t) = \sum_{n=1}^{\infty} a_n(t) \phi_i^{(n)}(\vec{x}) \]  

with

\[ a_n(t) = \int_D u_i(\vec{x}, t) \phi_i^{(n)}(\vec{x}) d\vec{x} \]  

Since the geometry of the axisymmetric sudden expansion is in cylindrical coordinates, the current analysis is expressed using the notation of figure 2, where \((x_1, x_2, x_3) = (z, r, \theta)\).
An appealing characteristic of this axisymmetric flow, is that the azimuthal direction is periodic, symmetric and invariant in azimuth, in an average sense. Where the later is concerned, this was checked at two different radial locations and is shown in figure 3. Here, \( U(r, z) \) and \( \langle \sigma_u(r, z)^2 \rangle \) have been averaged over azimuth (where \( \theta' = \theta_0 + \Delta \theta \)) and show that the aberration of the streamwise mean and Reynolds stresses are within 10%. Hence, with eigenfunctions that are harmonic in azimuth, the POD can be simplified by performing a Fourier cosine transformation of the original kernel \( R_{ij} \) to yield a new kernel \( B_{ij} \) as a function of Fourier-azimuthal mode:

\[
R_{ij}(r, r', \Delta \theta, z) = \sum_{m=0}^{\infty} B_{ij}(r, r', m, z) \cos m\theta
\]  

(2.5)

where for \( m = 0 \),

\[
B_{ij}(r, r', 0, z) = \frac{1}{\pi} \int_{0}^{\pi} R_{ij}(r, r', \Delta \theta, z) \, d\theta.
\]  

(2.6)

and for \( m > 0 \) otherwise,

\[
B_{ij}(r, r', m, z) = \frac{2}{\pi} \int_{0}^{\pi} R_{ij}(r, r', \Delta \theta, z) \cos m\theta \, d\theta.
\]  

(2.7)

Here \( z \) is used to designate a particular streamwise position, and \( \Delta \theta \) as the azimuthal separation between \( \theta_0 \) and \( \theta' \). The transformation to a Fourier-azimuthal mode number, \( m \), using (2.6) and (2.7) results in an altered form of (2.1),

\[
\int_{D} B_{ij}(r, r', m, z) \phi^{(n)}_{ij}(r', m, z) r' \, dr' = \lambda^{(n)}(m, z) \phi^{(n)}_{ij}(r, m, z)
\]  

(2.8)

Further implementation of these low-dimensional tools have been shown by Tinney et al. (2002a), using the Complementary Techniques of Bonnet et al. (1994), to reconstruct an estimate of \( u_i(x, t) \) from (2.4) via Linear Stochastic Estimation.
3. Experiment

The axisymmetric sudden expansion has been an on-going research effort of Glauser and colleagues for several years. The original facility was constructed at Clarkson University in 1989 to study turbulence closure schemes with results obtained with LDA measurements. Cole & Glauser (1998a) later updated the facility to include the flying hot-wire system described in § 3.1. Research efforts continued with Eaton (1999), as well as studies by Tinney et al. (2002b). Figure 4 depicts the axisymmetric sudden expansion facility located in the College of Engineering at Clarkson University.

Air-flow is provided to the tunnel by an axial blower powered by a three-phase 7.5hp motor. The air enters through an axisymmetric linear diffuser where it is sent through honeycomb flow straighteners, and two sections of fine mesh grid. The flow straighteners are essential for removing any swirling motions induced by the axial blower. Air then travels through a matched 5\textsuperscript{th} order contraction (ratio of 14:1), and into a 3\textit{inch} diameter plexiglass pipe, 12\textit{ft.} in length. The length to diameter ratio of the pipe (48:1) ensures fully developed flow at the inlet to the expansion test section. The expansion ratio of the test section is 3, (3\textit{in}. diameter pipe to 9\textit{in}. diameter pipe), and is 5\textit{ft.} in length. A small slot (0.375\textit{in}. wide by 46.5\textit{in}. long) was milled along the bottom surface so
that a probe could be attached to a flying traversing mechanism located outside the expansion’s test section. The traversing mechanism sits on the top surface of a bench and runs parallel to the axis of the tunnel. Two retractable tape measures are attached to the base of the probe sting to prevent air from exiting the tunnel through the slot. An exit section comprising of 0.25in. diameter drinking straws, sandwiched between two fine mesh grids, is incorporated to prevent any downstream disturbances from influencing the flow through the test section.

Special attention was given to the alignment of the sled traverse, with respect to the tunnel’s axis, so that the vertex of the probe rake was located in the center of the tunnel during its entire streamwise displacement. Before the experiment, a cross-wire probe was positioned at thirteen evenly-distributed locations ($\Delta r = 3\text{mm}$) at the expansion’s inlet, to determine the inlet profiles along four planes, ($0^\circ, 90^\circ, 180^\circ, \& 270^\circ$). Details about the data acquisition system and instrumentation, are discussed in § 3.2. These
profiles were created by ensemble averaging over five statistically-independent blocks (2,048 samples/block) sampled at 100Hz, and are shown in figure 5 to collapse extremely well, given the statistical uncertainty of the measurements (≈3%). Based on the solution given by Blasius where $U = 0.99U_{cl}$, the average boundary layer at the inlet was found to be 7.06mm, or 0.093D thick. $D$ for this calculation is the diameter (3in.) of the smaller pipe at the inlet to the expansion, and the inlet mass flow rate, expressed as a function of the centerline velocity was calculated to be $4.7E^{-3}U_{cl}$.

Mean radial ($V/U_{cl}$) and azimuthal ($W/U_{cl}$) velocities were measured along four orthogonal planes, as a function of radial distance at the inlet, and are shown in figure 6. These profiles are essentially zero, and ensures no appreciable swirl at the inlet to the sudden expansion. Measurements were conducted at a Reynolds number of 54,000 based on inlet centerline velocity ($U_{cl} = 10.35ms^{-1}$) and inlet pipe diameter.

3.1. Flying-Wire

Due to the directional ambiguity of hot-wire anemometry, the flying-wire technique was utilized to capture the two-point statistics in regions where reverse flow and hot-wire
rectification errors were likely. This technique has been well documented. Watmuff et al. (1983) constructed an air bearing system which provided nearly frictionless motion to a moving sled of hot-wire probes used to capture wake profiles behind three-dimensional bluff bodies. Panchapakesan & Lumley (1993) used a similar shuttle system capable of moving at $2\, \text{ms}^{-1}$, to accurately acquire the triple moments in the free shear layer downstream from a 10 cm jet. This was necessary, as it had been shown that stationary CTA measurements of the jet’s turbulent shear layer were significantly different when compared to LDA measurements. Thus, the motivation for employing such a technique is to impose a large bias velocity on the probe’s sensing-wire, in an attempt to improve the linearity of the probe’s response. Furthermore, the cone angle between a cross-wire probe’s axis and the oncoming velocity vector can be reduced, thereby increasing the accuracy of measurements in high turbulence intensity flows, or flows with unsteady recirculations, as is the case here.

In the present study, a stepper motor was utilized to accelerate and decelerate a sled (that the probe-rake was attached to) to a prerecorded impulse velocity near $3.5\, \text{ms}^{-1}$ between $z/h = 14$ and $z/h \approx 0$. The sled was mounted to two linear rails to ensure
precise, controlled movement along the streamwise direction. A close-circuit belt acted as the drive shaft between the sled and the stepper motor. During each sled pass, the position of the stepper motor was recorded simultaneously with the CTA’s output so that the response of probe array’s output could be properly associated with their streamwise positions in the flow.

The mean impulse velocity of the sled, shown in figure 7a, was measured prior to the experiment using a single-wire probe averaged over 1,024 runs. This was acquired without flow moving through the expansion, and was later subtracted from measurements where the tunnel was turned on. Flying-wire measurements were then repeated with the tunnel turned on (flow moving through the expansion test section), and were compared with stationary measurements under the same flow conditions, in an attempt to reconcile any differences between the two techniques. The probes were situated on the centerline, since flow reversal is not present at this location. Excellent accordance can be seen between the two data-sets shown in figure 7b, despite small variations in the averaged motion profile of the sled.

Mean streamwise velocity profiles \( \frac{U(r)}{U_{cl}} \) in figure 8a are shown for several radial positions. From this, one can see that the recirculating features near the outer wall region of the flow at \( z/h = 6 \) have been effectively sensed. A discussion by Cole & Glauser (1998a) provides a more comprehensive outline of the flying-wire system located in the sudden expansion facility.

Due to the physical nature of wall-bounded flows, the principle of mass conservation, shown in figure 8b using the measurements shown in figure 8a, was calculated using a trapezoidal substitution of (3.1).

\[
\dot{m} = 2\pi \rho \int_0^R U(r) r dr
\]  

(3.1)

In general, the mass flow rate is slightly lower before and after the reattachment
Figure 7. (a) Average motion profile of flying-wire traverse using single-wire probe. (b) Comparisons between stationary and flying-wire probe on the centerline.

Figure 8. (a) Mean streamwise velocity profiles, where $U_{cl} = 10.35 \text{ms}^{-1}$. (b) Mass flow rate calculated with the single cross-wire data.

region (near $z/h = 8$ & 9); presumably the synthesis of small statistical errors in the measurements and numerical integration error. Had it been flow leakage, one would expect to see a lower mass-flux at the downstream location. Cole (1996) investigated this in more detail and found many qualitative similarities with other investigators.
The cross-wire measurements incorporated sixteen differential channels of thermal anemometry, each sampled at a frequency of $2kHz$ and low-pass filtered at $820Hz$. The eight probes were custom built Auspex cross-wires, with tungsten wire sensing-lengths of $1.0mm$ and diameters of $5\mu m$ (aspect ratio of 200). Given the sensing-lengths of the wires, and the radial and streamwise variations of the mean velocities, the corner frequency $f_c = \frac{U(r,z)}{2l_w}$, ranged between $5.2kHz$ and $1.2kHz$ for the flying-wire measurements, and between $4.1kHz$ and $420Hz$ for any stationary measurements at the Reynolds number studied. This is based on a suggestion that the smallest resolvable structure is twice the sensing-length of the hot-wire probe. Thus, at low mean velocities, the probes themselves could act as low-pass filters.

Data acquisition was accomplished using IOTech Analog to Digital converters with simultaneous sample and hold. The sampling capabilities of the converters contain 16-bit resolution, yielding quantization errors on the order of $3E^{-4}$ volts for a $\pm 10$ volt bipolar range. Calibration of the cross-wires used an expression relating the free-stream velocity, $U_o$ to the effective velocity, $U_{eff}$ (seen by the probe) as suggested by Hinze (1975),

$$U_{eff,i}^2 = U_o^2(\cos^2 \phi_i + k_i^2 \sin^2 \phi_i)$$  \hspace{1cm} (3.2)

where the indices, $i = 1, 2$ refer to probe wires. Using trigonometric substitutions and other manipulations based on the cross-wire geometry, the relationship in (3.2) is reduced to the expressions: $U_1 = U_o \cos \beta$ and $U_2 = U_o \sin \beta$, to determine the individual velocity components, $U_1$ and $U_2$. Here, $\beta = \phi - \frac{\Phi}{2}$, and $\Phi$ is a constant geometric property of the hot-wire and is equal to $90^\circ$, as was demonstrated by Cole (1996).
Figures 9 and 10 show the normal and shear stress profiles obtained from 450 statistically independent sled passes, using a single radial rake of cross-wire probes (eight probes) and the flying-wire technique. In these figures, an imaginary extension of the expansions’s lip-line and the mean reattachment point is drawn. As one can see, the maximum values of \( u'^2 \) are shown to comprise the greatest amount of turbulent energy, while \( v'^2 \) and \( w'^2 \) are maximally similar. All of them peak before the reattachment point near \( z/h = 7 \), and fall rapidly (nearly half of their energy is lost), by \( z/h = 10 \). The streamwise/azimuthal shear stresses \( uw \) are not shown, as they are insignificant in value.

What is striking about these normal stresses is that until the collapse of the potential core, the location where the turbulent energy is maximum is favorable to the irrotational regions of the flow, rather than the expansion lip-line. Gould et al. (1990) and Cole & Glauser (1998a) demonstrated nearly identical features, despite differences in the inlet conditions. In the former investigation, a two-component LDV system (with correcting lens) captured the axial, radial and shear stresses along with the triple products, all of which were compared with a \( \kappa - \epsilon \) turbulence model. Though the turbulence model slightly over predicted the shear stress values in the shear layer and in the recirculating region, the divergence of the shear layer towards the potential core (before its collapse), was evident. This is surprising considering the profiles of Castro & Haque (1987), who showed that the shear layer center-line in the separated region behind a flat plate normal to an airflow, arched towards the outer wall when normalized by a standard reference velocity \( U_{cl} \). They attributed this augmentation to the mean streamline curvature; a known phenomenon by which a slightly lower pressure in the recirculating region forces the mean shear layer to curve more rapidly towards the wall than in other free shear flows.
(i.e. the behavior of the axisymmetric jet would be the most appropriate comparison to the current study). Though the normal stresses in figures 9 and 10a demonstrate similar behaviors (in magnitude) to those of Bradshaw & Wong (1972) and Castro & Haque (1987), the differences in the location of peak energy bequest a deeper understanding of the effect of the streamline curvature on this flow.

Castro & Haque (1987) showed that the normal stresses, as well as the vorticity’s thickness, were typically larger in the early part of the flow when compared to the planar turbulent mixing layer. Their findings also demonstrated an overcoming of the turbulent spanwise energy relative to the normal energy, and can be interpreted in concert with the current stress profiles illustrating an increase in the normal azimuthal stresses near the wall regions of the flow, where as the radial stresses continue to decay due to the wall’s impermeability. Since $U_\infty$ is constant over all axial positions in the flow of the planar backstep flow, a self similar study necessarily employs the vorticity thickness and is ideal in the setting described by Castro & Haque (1987) when compared to the planar mixing layer. However, in the axisymmetric sudden expansion, the residuum of a potential core (specifically its collapse around $z/h = 5$), induces a dwindling of the centerline statistical features which refuse a linearly self similar solution to the profiles. Therefore, the range over which a direct comparison of the axisymmetric jet to the current study is limited to a region that does not include many of the features of the flow that are of interest, specifically the reattachment region around $z/h = 9$.

The stress profiles shown here are perhaps one of several marque differences between axisymmetric sudden expansion flows, axisymmetric jets, and the planar backstep / mixing layer studies. Though the axisymmetric sudden expansion possesses emblematic features of the axisymmetric jet (a potential core region), and of the planar backstep flow (streamline curvature), the flow through the axisymmetric sudden expansion produces
Figure 9. The Reynolds stresses (a) $<u^2>/U_{cl}^2$, (b) $<v^2>/U_{cl}^2$.

a turbulence structure that is nearly innate to both, where self-similarity in concerned.
We will revisit this discussion in § 5.

4. Multi-Point Cross-Wire Measurements and the Two-Point Statistics

The multi-point azimuthal spatial measurements of the flow incorporated a probe sting consisting of two radial rakes of cross-wire probes mounted on the traverse’s sled. The
Figure 10. The Reynolds stresses (a) $<w^2>/U_{cl}^2$, (b) $<-uv>/U_{cl}^2$.

The probe rake used here differs from the rake used to obtain the results presented in § 3. One radial rake was fixed while the other rake had the freedom to pivot between 0 and 180 degrees, relative to the other rake. A grid density of comparatively high resolution is shown in figure 11, and illustrates the 8 radial and 45 azimuthal locations (360 points) used to capture the two-point statistics of the flow. An azimuthal separation of $8^\circ$ was measured using a compact protractor, and the radial increments were fixed in the probe array’s design: $r/R = 0.12, 0.23, 0.35, 0.46, 0.57, 0.69, 0.81, & 0.92, (\Delta r/R = 0.115$
and $R = 4.5\text{in.}$ is the expansion’s radial distance). The cross-wires were oriented to measure the streamwise $u$, and radial $v$ components of the fluctuating velocity in order to construct the normal and shear stress terms ($uu$, $uv$, $vu$ and $vv$) of the 2-component kernel in (4.1). The correlation at $r/R = 0$, was not obtained because of the design of the probe rake.

$$R_{ij}(r, r', \Delta \theta, z) = \langle u_i(r, \theta_o, z)u_j(r', \theta', z) \rangle$$ (4.1)

The mean azimuthal invariance of this flow imposes a symmetry condition that should be exercised, on account that the relationship between $\theta'$, incident to the same radial and azimuthal origin $\theta_o$, is the same, if true statistical convergence has been achieved. In doing so, the experimental time can be reduced proportionately by a factor of 2. Thus, the correlations obtained over the plane $\theta = 0^\circ$ & $180^\circ$, were mirrored onto the plane $\theta = 180^\circ$ & $360^\circ$, thereby reducing the number of grid points from 360 to 176.

These experiments involved three unique probe arrangements (figure 12) in order to capture the correlations as a function of azimuthal separation. For each arrangement, the correlations were obtained by sweeping Rake 2 from $8^\circ$ to $176^\circ$ relative to Rake 1, while holding Rake 1 stationary. The exception was in the third arrangement (figure 12c) where Rake 2 started at $0^\circ$ relative to Rake 1. Using the symmetry condition, the selection of the top probe Rake in the third arrangement was arbitrary. Each arrangement consisted of 450, statistically independent sled passes. 2,048 incremental samples were acquired during each sled pass as the cross-wires were propelled down the tunnel between $z/h = 14$ and $z/h \approx 0$. Therefore, based on the Reynolds stress values shown in § 3.3 and substituting $U_{cl}$ in terms of $U/U_{cl}$, the percent variance from the mean, using $\epsilon = \sigma_u(U\sqrt{N})^{-1}$ and $N=450$, was less than 1% in the core, and %5 at the outer wall.
Figure 11. Grid Density of 8 radial and 45 azimuthal locations for capturing the two-point statistics.

Figure 12. Probe arrangements (a) 1, (b) 2, (c) 3.

4.1. Correlations

Several of the correlations were selected to provide the reader with a general picture of the statistical characteristics of the turbulent motions of this flow. Figure 13 illustrates
Figure 13. The cross correlation (a) $R_{uu}(r, r', 0, 6)$, (b) $R_{vv}(r, r', 0, 6)$, (c) $-R_{uv}(r, r', 0, 6)$.

The normal and shear stress velocity components, $R_{uu}(r, r', 0, 6)$, $-R_{uv}(r, r', 0, 6)$ and $R_{vv}(r, r', 0, 6)$ obtained within the reattachment region where the fluctuations were found greatest. It is clear the axial component of velocity is correlated over nearly the entire shear layer. A modicum of the full statistical field, it is these quantities from which the POD’s basis functions are derived.

In figures 14 and 15 the radial and azimuthal characteristics of the correlation function $R_{uu}(r, r', \Delta \theta, z)$ are illustrated by fixing $r$ and $\theta_o$, while moving $r'$ and $\theta'$. This is shown at four streamwise positions in the flow: $z/D = 3, 6, 9, 12$. The scaling in these figures are identical so that the relative strengths can be compared.

In figure 14a, the correlation tensor $R_{uu}(0.35, r', \Delta \theta, 3)$ clearly illustrates the initial development of events shedding from the expansion’s lip. Given the slow roll-off of $R_{uu}(0.35, r', \Delta \theta, 3)$, which is near the center of the shear layer and close to the lip where one would expect to find a large amount of energy in the higher azimuthal modes (if it did exist), it is clear that the dominant length scales in the flow are sufficiently large. Thus, any scales which may have been aliased because of the spatial distances between probes, has had a negligible effect on the results. The fact that the topography of the correlation at $r/R = 0.69$ in figure 14b lacks any significant features, insinuates a relatively dead zone in the flow. Subsequently, sub-figures 14c, & d manifest increases in
Figure 14. The cross correlation (a) $R_{uu}(0.35, r', \Delta \theta, 3)$, (b) $R_{uu}(0.69, r', \Delta \theta, 3)$, (c) $R_{uu}(0.35, r', \Delta \theta, 6)$, (d) $R_{uu}(0.69, r', \Delta \theta, 6)$.

Coherence at $r/R = 0.35$ and $0.69$. Closer observation indicates that the peak is taller and narrower at $r/R = 0.35$ than at $r/R = 0.69$, however the azimuthal length scale is shown to encompass a much broader area at the higher radial position. Similar findings to this are shown at $z/h = 9$, (figure 15a & b) characterizing the radial and streamwise growth of the turbulent shear layer. In the redeveloping region of the flow at $z/h = 12$ (figure 15c & d), a decadence of the radial correlation reciprocates the broadening of the azimuthal length scales.

Physically, one may think of the two-point correlation as a measure of the strength
of an eddy and the coherent repeatability of its motions, on average, through a fixed space. Therefore, as the flow develops in the axial direction, it would be expected that the azimuthal length scale should grow. This is indeed the case, as these illustrations have shown, i.e. the azimuthal correlation’s length does not decay, but rather grows with increasing distance from the expansion lip.
5. Results of the Modal Analysis

The eigenspectra, $\lambda(n)(m, z)$, from (2.8) are shown to provide information about the relative weight of the intrinsic turbulent motions to the overall turbulent flow-field. Since the POD and Fourier techniques are decompositions of energy, the total $TKE$ (figure 16), is compared to a full summation of the eigenmodes (equation 5.1), thus demonstrating the axial location (around $z/h = 8$) where the turbulent activity is greatest.

$$\zeta(z) = \sum_n \sum_m \lambda(n)(m, z)$$  \hspace{1cm} (5.1)

A more detailed depiction of the $TKE$ across the axial and radial plane is shown in figure 17, to make lucid, the location of greatest turbulent energy. The illustration employs $\kappa = \frac{2}{3}(\bar{u}^2 + \bar{v}^2)$, where the coefficient for $\kappa$ was determined by Cole & Glauser (1998a) and was based on a discussion by Stieglmeier et al. (1989), who showed that the typical value ($3/4$) over estimated the $TKE$ when only two components were included. The overestimate assumed that $< w^2 >$ existed somewhere between $< u^2 >$ and $< v^2 >$. However, as figures 9b and 10a demonstrate, $< w^2 >$ and $< v^2 >$ are maximally sim-
Figure 17. The turbulent kinetic energy in the $r,z$ plane.

ilar, and agrees with the aforementioned investigations. These results show qualitative similarities to those of Cole & Glauser (1998a), and a consistency between direct measurements of the $TKE$ and the solutions from the decomposition. The expansion’s lip-line is identified in this illustration, and differs from the peak location of greatest energy. The kinetic energy balance of Cole & Glauser (1998a) showed that the turbulence production occurs within the first four step-heights, the diffusion of this energy primarily transpired along the outer regions of the flow near the wall, and the convection was manifest within a relatively dead zone in the flow within the recirculating region towards the expansion lip.

A plausible explanation for the offset of greatest turbulent energy, when compared to free shear flows (2-d mixing layers, axisymmetric jets), whereby the location of maximum shear coincides with the upstream lip-line with a slight deviation towards the low-speed side/entrainment region, is that the coalescence of the turbulence along the outer wall and re-circulating regions impel the center of the shear layer towards the core region of the flow. Thus, an aggregate swelling of the turbulence activity near the wall is the
constitute that effects the location of greatest turbulent energy. A detailed analysis of the eigenvectors in § 5.2 will justify this idea. This appears to be more aggressive, due to the wall-bounded nature of this flow, than the studies of Castro & Hahque (1987) (separated shear-layer behind a flat plate with a pseudo infinite free-stream) whereby the shear layer centerline was established to objectively juxtapose the plane mixing layer, and showed that the stresses peaked towards the low-speed side of the shear layer. In this later investigation, the stresses were normalized by the velocity difference $\Delta U = U_{max} - U_{min}$, the axially local maximum and minimum velocities, respectively.

The total energy from the radial decomposition is shown in figure 18 using (5.2). This is similar to the findings of Glauser & George (1987a), Ukeiley et al. (1999), Cole (1996), and others, whereby a large percentage of the local energy ($\sim 30\%$) is captured in the first radial POD mode alone. With the first two POD modes combined, nearly 50% of the energy is contained, and so on. A slight increase in the relative energy contained in the first (largest) POD eigenvalue implies that the development of the radial structure becomes more coherent as it convects downstream from the expansion lip. Since there is an abundance of radial energy in the first five POD modes, the eigenvalue distributions displayed in subsequent figures are simplified using only the first few radial modes.

$$\xi(n, z) = \sum_{m} \lambda^{(n)}(m, z) \zeta(z)$$  \hspace{1cm} (5.2)

5.1. Eigenvalue Distribution

The eigenvalue distributions are shown in figure 19, normalized by (5.1). Upstream towards the expansion lip ($z/h = 3$ in figure 19a), the distribution of the Fourier-azimuthal energy is fairly broadband in contrast to a coalescence of energy in the lower Fourier-azimuthal modes shown in successive figures. This is partially attributed to the fully developed inlet conditions to the expansion, where Cole (1996) has shown that the tur-
bulent spectral densities are broad. As the flow evolves downstream, the integral scales increase, and the energy becomes less broad and more low-dimensional in azimuth. The authors found no dramatic change in the eigenspectra beyond azimuthal mode $m = 6$ for all step-heights, with a preponderance of energy in the first few Fourier-azimuthal modes, primarily $m = 0, 1, 2 & 3$. This suggests that there was a diminutive level of aliasing of the azimuthal decomposition as described by Glauser & George (1992), and that the resolution of the azimuthal grid exceeds what was necessarily required to resolve the Fourier-azimuthal spatial modes in this flow. These results, interpreted in concert with the decomposition’s rapid convergence, demonstrate the low-dimensional characteristics of this flow.

In figure 19a (near the expansion inlet at $z/D = 3$), the eigenvalue distribution illustrates an ascendancy of the $m = 2$ mode, relative to the overall local energy. The $m = 1$ mode, which is typically responsible for flapping, is shown to possess nearly congruent energy in relation to the next odd mode ($m = 3$). Note, as is shown in figures 19b ($z/D = 5$) and 19c ($z/D = 7$), the eigenvalue distributions are similar to those in fig-
Figure 19. Eigenvalue distribution, POD modes 1:5, azimuthal modes 0:11, (a) $z/h = 3$, (b) $z/h = 5$, (c) $z/h = 7$, (d) $z/h = 9$, (e) $z/h = 11$, (f) $z/h = 13$. 
This distribution among the first few Fourier-azimuthal modes is most likely a construct of the fully developed inlet conditions suggested earlier, and the re-entrainment of unsteady fluid through the recirculating region. This differs drastically from the modal distribution shown by Glauser & George (1987a), Citriniti & George (2000) and Gamard et al. (2002) in the axisymmetric turbulent mixing layer studies, which have been shown to possess dominance in the first symmetric $m = 0$ mode, followed by higher modes 4, 5 \& 6. Bradshaw \& Wong (1972) suggested a more detailed explanation, in that the shear stress features $< uv >$ in the shear layer were augmented by the re-entrainment of a stress-bearing fluid, and was typically $(1 + 0.2)^2$ times larger than the values found in mixing layer investigations, even when the influence of the initial boundary layer was negligible. This was quite surprising since the mean re-entrainment flow never exceeded 0.2 times the plane shear layer values. Castro \& Haque (1987) illustrated the same features and showed that up until reattachment, the shear stress levels exceeded those of the plane mixing layer, along with a strong increase (by a factor of 2) by reattachment when normalized by $\Delta U$. Castro \& Haque (1987) also suggested that differences in amplitude between the axial and normal stresses was evidence of flapping. Though we have shown that this region of the flow (in the axisymmetric sudden expansion) is dominated overall by the $m = 2$ mode, the $m = 1$ mode still sustains a large portion of the turbulent energy through this region. However, an unravelling of the axisymmetric geometry into a 2-d planar geometry, would transpose the azimuthal modes to spanwise modes, thus illustrating evidence of a flapping structure. If one considers that the pressure strain distribution of the flow through a completely wall bounded axisymmetric geometry has more influence on the entire field because of the diametric disposition of the structure, than in the 2-d backstep geometry where the free-stream is "infinite", this does impose a qualitative similarity between the two scenarios.
Within the reattachment region at $z/D = 9$ (figure 19d), the flow undergoes a pronounced modal switch from the $m = 2$ Fourier-azimuthal mode, to the flapping mode ($m = 1$). This is in the region where a consequence of the walls impermeability is more apparent. Bradshaw & Wong (1972) observed the creation of a “unique” structure at reattachment caused by a change in the shear layer’s mass flow. Despite little changes in the velocity gradient ($\partial U/\partial r$), they proposed a more practical explanation for the rapid decay in the Reynolds shear stresses in that the impermeability of the wall caused the absence of a normal velocity component, ($v = 0$). This earlier investigation contained limited information regarding the development and evolution of the spanwise structure and yet concluded that the dominant mechanism at reattachment was a bifurcation of the structure. In either regard, the decay of the shear stress terms has been observed in many reattaching flows [Kim et al. (1978), Chandrsuda & Bradshaw (1981), Baker (1977), Castro & Haque (1987) and Eaton & Johnston (1981)], regardless of large differences in the initial conditions, and has been generally attributed to the mean streamline curvature. Castro & Haque (1987) further cited that while the axial and normal stresses increased towards reattachment with an abrupt decay thereafter, the spanwise (azimuthal in our study) stresses continued to rise only to be constrained by the pressure strain relationship.

In the backward facing step, with a pseudo-infinite free stream above the shear layer, a different transformation of energy may be apparent in that some portion of the normal energy, that which is not consumed by recirculation, is deflected axially and in the cross-stream direction (towards the free stream region), thus passively increasing the height of the boundary layer. The measurements presented by Chandrsuda & Bradshaw (1981) and Castro & Haque (1987) show this behavior where the shear layer displays a small abrupt heightening after reattachment, followed by a decrease in the surface pressure
C. E. Tinney, M. N. Glauser, E. L. Eaton, J. A. Taylor
distribution, although was not discussed in detail. The heightening of the shear layer
quickly stabilizes towards a more shallow rate of growth in the relaxation regions of the
flow.

A glance at figure 20 illustrates the surface pressure distribution \((C_p = p - p_{ref}/\frac{1}{2}\rho U^2)\)
from Chandrsuda & Bradshaw (1981), along with wall pressure measurements in the
current facility. The abscissa and ordinate axes in this figure are normalized by the mean
reattachment length \(z_r\), and the peak pressure value \(C'_{p}\), respectively, to better illustrate
their qualitative similarities and differences. The value for \(z_r\) from the measurements
of Chandrsuda & Bradshaw (1981) was about 5.9 step-heights. In both instances, the
recirculating region is clearly illustrated by the mean pressure deficit, and the peak
pressure value is located after reattachment. However, while the surface pressure in the
backward facing step continues to fall after its maximum, the wall pressure in the current
study appears to have asymptotically stabilized (in the regions measured) because of the
outer walls restraint on the boundary layer’s development.

Further downstream in the redeveloping region of the flow (figure 19f), the flapping
mode continues to dominate the modal behavior of the large scale motions with a sig-
nificantly increasing contribution from the axisymmetric mode. Even though the present
investigation is limited to \(z/h = 13\), one could conjecture a more important contribution
from the \(m = 0\) mode asymptotically as the flow evolves to its fully developed state far
downstream.

5.2. Modal Reconstruction of the Kernel

To proceed with a more detailed explanation of these findings about the turbulent/modal
behaviors of this flow, the eigenvectors from (2.8) are used to reconstruct the kernel \(B_{ij}\)
with \(r = r'\) for various azimuthal mode numbers. Only the first two POD modes (\(~ 50%\)
of the turbulent energy) will be used in the reconstructions, since additional POD modes
Confining our discussion to the axial component of velocity (throughout the remainder of the discussion, we will refer to this as simply 'energy'), it is clear that the axisymmetric mode (figure 21a) possesses the dominant modal behaviors over all other modes, near the outer recirculating wall regions of the flow leading to reattachment. Being described by Jung et al. (2004) as a spatially convected disturbance, perhaps typifies the feedback mechanisms that are responsible for ensuing higher modes along the shear layer regions of the flow. We will give more detail to the discussion of the feedback mechanisms in § 5.3. However, following the collapse of the potential core, the energy of the axisymmetric mode is shown to be spatially concentrated to the central regions where the flow state commences towards a more fully developed profile. Though the axisymmetric mode ap-
pears to follow the center of the shear layer in the early part of the flow \((z/h = 3 \text{ to } 7)\), it is the \(m = 1\) mode that emerges where the axisymmetric mode decays \((z/h > 7)\). This increasing importance of the \(m = 1\) mode, emerging where the potential core collapses \((z/h = 5 \text{ to } 9)\), is similar in behavior to what is found in the axisymmetric jet studies of Jung et al. (2004), whereby the higher low-dimensional Fourier azimuthal modes, particularly \(m = 2\), have been found to proceed the merging of the inner shear layers. Perhaps due to the stabilizing nature of the streamline curvature in the axisymmetric sudden expansion, it is the helical mode that gives rise after the collapse of the potential core, as opposed to the higher Fourier-azimuthal modes (Jung et al. (2004) and Glauser & George (1987a)).

Interestingly, the energy reconstructions using the \(m = 2\) and \(m = 3\) modes (figures 21c and d respectively), are shown to cover the entire spatial distribution of the shear layer (refer to the axial Reynolds stress in figure 9a). Thus, in a feedback sense, they are acting to transfer energy from the spatially convected disturbance, that is the \(m = 0\) mode along the outer recirculating wall regions, to the center of the shear layer. Even more intriguing are the reconstructions of the \(m = 4\) and \(m = 5\) modes, shown in figures 21e & f, respectively. These manifest the streamline curvature of the flow, a consequence of the adverse pressure effects of the expansion, and whose amplitude rapidly decays as they approach the wall. This suggests that the energy either (1) dissipates, is (2) rapidly transferred elsewhere, or plausibly, some combination of both. In view of the axisymmetric mode’s sudden rise at the wall near reattachment (figure 21a) insinuates that the energy (in the higher modes) transfers to the lower modes because of the wall’s impermeability (i.e., an inverse cascade from higher Fourier-azimuthal modes 4 and 5 to mode 0).

Furthermore, the pronounced disposition of the higher Fourier-azimuthal modes along
the streamline curvature of the shear layer suggests that the turbulent features of the flow through the axisymmetric sudden expansion are necessarily unstable to the higher Fourier-azimuthal modes. The decadence of all other higher modes \((m > 5)\) after reattachment, is symbolic of the behavior of the \(m = 5\) eigenvector reconstruction shown in figure 21f.

5.3. A Model for Feedback

An interpretation of the results presented in the previous sections suggests the following plausible mechanism for the feedback due to the recirculating zone. It is important to first point out that where comparisons to the axisymmetric jet’s near-field shear layer seam feasible \((i.e., \ z/h \leq 5)\), the eigenvalue spectra (figures 19a and b) demonstrate noticeably different behaviors than the jet, as was discussed in § 3.3. In particular, Fourier-azimuthal modes 2 and 3 (in the axisymmetric sudden expansion) contain substantially more relative energy, whereas modes 4, 5 and 6 contain less \(^*\). Thus, the suppressed turbulent development of the shear layer is clearly attributed to the stabilizing effects of the streamline curvature (Castro & Haque (1987)), which is an interesting effect that the outer wall has on the flow.

In view of the energy exchange at reattachment in § 5.2, the rapid decay of the higher Fourier-azimuthal modes \((m = 4 \text{ and } 5\) in figures 21e and f, respectively) are shown to transfer some, if not most, of their energy back upstream. That which is recirculated upstream along the outer wall, is transferred spatially into the form of an axisymmetric.

\(^*\) The modal decompositions of the axisymmetric jet by Glauser & George (1987a), Citriniti & George (2000), Jung et al. (2004) and Ukeiley et al. (1999) have all demonstrated that the pronounced modal event in the near field shear layer (absent of the column mode) were higher Fourier-azimuthal modes 4 through 6, that stabilized to mode 2 after the collapse of the potential core. Gamard et al. (2002) recently showed that this \(m = 2\) mode also dominated the transitional and far field regions of the jet, \(6 < z/D < 69\).
Figure 21. The eigenvector reconstruction of $B_{ij}$ with one POD mode for several Fourier-azimuthal modes (a) $B_{11}^{1+2}(r, 0, z)$, (b) $B_{11}^{1+2}(r, 1, z)$, (c) $B_{11}^{1+2}(r, 2, z)$, (d) $B_{11}^{1+2}(r, 3, z)$, (e) $B_{11}^{1+2}(r, 4, z)$, (f) $B_{11}^{1+2}(r, 5, z)$. 
ric mode, hence the relatively high contribution to the spatial distribution of mode 0 at the wall up until reattachment. This is a clear confirmation of the observations made by Bradshaw & Wong (1972) and Chaturvedi (1963) regarding the creation of a "unique" structure at reattachment, shown here as an inverse cascade of energy from higher modes to lower modes initiated at the wall. However, the absence of any recognizable modal energy after reattachment along the outer wall regions of the flow suggests a clear absence of coherent structures. In as much as the hypothesis by Kim et al. (1978) suggests that some eddies at reattachment are swept upstream and some downstream, it is highly unlikely under the conditions investigated. Therefore, the allotment of coherent turbulent energy from these higher modes (4 and 5), that which is not entrained by the adverse pressure effects of the outer wall, amalgamate into low-ordered modal energy (0 and 1) as they convect downstream along the center of the pipe, where the dominant mechanism that is shown leading into the redeveloping region of the flow in figure 19f, is helical/flapping.

The difference (in the Fourier-azimuthal mode number spectra) between jet flows and sudden expansion flows, before the collapse of the potential core ($z/h < 5$), particularly the greater role of modes 2 and 3 in the axisymmetric sudden expansion, implies an important role of the feedback mechanism. Based on the observations made thus far, it is possible that there is a direct transfer of energy from the higher modes ($m = 4$ and 5) to the lower modes ($m = 2$ and 3) via the feedback mechanism ($m = 0$) in the recirculating wall region. Figure 22 illustrates this feedback mechanism in summary to figure 21. Since the spatial distribution of the energy in modes 2 and 3 possess substantial energy up to the wall through the recirculating and reattachment zones, we can think of the feedback mechanism ($m = 0$) as a pump, dumping energy from the higher modes (4 and 5 along the streamline curvature) to the lower modes 2 and 3 in the outer near-
wall regions of the shear layer. The above explanation accounts for the relatively higher energy in Fourier-azimuthal modes 2 and 3 when compared to the jet. Modes 2 and 3 then transfer their energy back into modes 4 and 5 where they follow the streamline curvature of the shear layer, and the feedback cycle is complete. The energy that is not recycled by the feedback pump, continues to convect downstream (mode 2) where it eventually transforms to lower-order structures (modes 0 and 1).

6. Conclusion

The decomposition of the streamwise and radial components of the fluctuating velocity field of the flow through an axisymmetric sudden expansion was investigated using Proper Orthogonal and Fourier Decomposition in radius and azimuth, respectively. The decomposition was performed along slices of the $r$, $\theta$ plane, at several streamwise positions, starting from the recirculating region ($z/h = 3$ to 7), through the reattachment region ($z/h = 7$ to 9), and into the beginning stages of the redeveloping region ($z/h = 9$ to 13), of this flow. The experimental methods employed a flying-wire technique which was shown to effectively capture the multi-point statistics using cross-wire CTA tools in a flow characterized by regions of high turbulence intensities, and flow recirculations.
The eigenvalues from the decomposition have brought to light the dominance of the Fourier-azimuthal and POD modes that govern a moderate to large percentage of this highly unsteady wall-bounded flow. The results of the radial decomposition using POD showed that approximately 30% and 20% of the total energy was contained in the first and second modes, respectively, through all streamwise positions studied. The Fourier-decomposition of the azimuthal structure into azimuthal modes indicated a noticeable shift in the energy among modes near reattachment, from a dominant $m = 2$ through the recirculating region, to a dominant $m = 1$ in the redeveloping regions of the flow. Summarizing the axial and radial distribution of the modal reconstructions using the axial velocity component of the kernel has provided a plausible model for interpreting the feedback mechanisms responsible for the low-ordered behavior of this flow. This feedback mechanism appears to transport turbulent energy from the reattaching shear layer at the wall, in the form of an axisymmetric ($m = 0$) mode through the outer recirculating wall regions of the flow. The energy that is transported excites the outer regions of the shear layer which evolve into Fourier azimuthal modes 2 and 3. Fourier-azimuthal modes 2 and 3 are argued to be both obtaining energy directly through the feedback as well as through a cascade from modes 0 and 1 downstream from reattachment. The driving mechanisms for the feedback are the higher modes $m = 4$ and 5, feed by modes 2 and 3 on the low-speed near wall regions of the shear layer, and are clearly shown to follow the streamline curvature of the flow to feed the axisymmetric mode at reattachment, thereby completing the feedback cycle.

Far downstream in the redeveloping region of the flow, and beyond the point of focus of the current investigation, it is believed that the symmetric azimuthal mode will eventually dominate the flow. This is based on an observation of the growing trend observed in the low dimensional azimuthal mode’s streamwise evolution. A future investigation of this
redeveloping region may prove this postulation true, and may also provide insight into the azimuthal structure at the inlet to the expansion, as they share similar Reynolds numbers and flow states.

REFERENCES


Taylor, J. A., Ukeiley, L. S. & Glauser, M. N. 2001 A low-dimensional description of the


