Abstract

The present study is focused on the development of an empirical low-order dynamical system (LODS) of a Mach 0.6 high-speed axisymmetric jet to be ultimately used for closed-loop flow control. An identification method is implemented to solve for the coefficients of the ordinary differential equation (ODE) describing the evolution of the flow. This ODE is derived from a Galerkin projection of the Navier-Stokes equations onto a basis of proper orthogonal decomposition (POD) eigenfunctions. An extensive database of the velocity and acceleration fields is therefore needed to be used as “training” data for the dynamical system. A dual-time particle image velocimetry (DT-PIV) experiment is designed and carried out to measure velocity and Eulerian acceleration in cross-flow planes from 3 to 10 jet diameters downstream. The setup comprises two stereoscopic PIV systems that sample velocity at two consecutive instants, the time separation being carefully chosen to resolve the scales of interest in the flow. POD is applied and the resulting low order dynamical system is exposed and its dynamics are validated against data previously measured in this flow. A preliminary experiment measuring and correlating the near-field to the far-field pressure is carried out to identify the most sound productive region in the jet, where the DT-PIV experiment focuses, and to give insight into the nature of the propagative sound sources. The results of this experiment lead to the understanding that the axisymmetric mode (azimuthal Fourier mode 0) of the near pressure field is the best propagator to the far-field, which can guide flow control strategies for noise reduction. With this result in mind, synthetic-jet based actuators are designed to be able to provide hydrodynamic perturbations at the nozzle exit to exploit the non-propagative nature of the higher azimuthal modes. The three main aspects of this work (dynamical system development, sound source identification and flow control device design) applied to the high-speed axisymmetric jet lay the preliminary grounds towards the implementation of practical closed-loop flow control for far-field noise reduction.
LOW-DIMENSIONAL TECHNIQUES FOR ACTIVE CONTROL OF HIGH-SPEED JET AEROACOUSTICS

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# Contents

List of Figures vii

Nomenclature xii

Acknowledgements xiv

Introduction 1

1 Background on jet noise identification and control 5
   1.1 Towards understanding jet noise 5
   1.2 The dynamics of the axisymmetric jet 8
   1.3 Jet noise control 11
      1.3.1 Passive techniques 12
      1.3.2 Active techniques 13

2 The Syracuse University large anechoic chamber and jet facility 16
   2.1 Skytop anechoic chamber 16
   2.2 High-speed jet facility 17
      2.2.1 Facility and flow conditions 17
      2.2.2 Measurements and data acquisition equipment 18
   2.3 Overview of the experiments 20
      2.3.1 Experiment 1: Near-field/far-field pressure measurements 20
      2.3.2 Experiment 2: Measurement of the Eulerian acceleration field 21

3 Relationship between the near- and far-field pressure 22
   3.1 Motivation 22
   3.2 Experimental setup 23
      3.2.1 Facility and flow conditions 23
      3.2.2 Measurements and data acquisition 23
   3.3 Results 26
      3.3.1 Near- and far-field pressure spectra 26
      3.3.2 Sound source location 30
      3.3.3 Modal description of the pressure 41
   3.4 Conclusions 47

4 Dynamical system of the loud region of the jet 49
   4.1 Measuring flow dynamics 51
   4.2 Dual-time particle image velocimetry instrumentation 54
      4.2.1 Jet facility and flow conditions 54
      4.2.2 Background on the technique 55
List of Figures

1.1 Laser-based visualization of the developing instabilities in the axisymmetric jet flow, Re=10,000. ........................................... 9
1.2 The four stages of turbulence production (left to right), model proposed by Glauser et al. [46] showing the break-up of a large coherent structure into higher order azimuthal structure due to the passage of an inner ring through the outer low momentum ring. .......... 10
2.1 Syracuse University large anechoic chamber and high-speed jet facility, with 6 far field microphones (in red circles). ................. 18
2.2 View of the location of the far-field microphones within the chamber, adapted from Tinney [102]. ............................................. 19
3.1 Picture and schematic of the near-field pressure circular holder. . . 24
3.2 Location of all far-field and near-field pressure measurement points. 25
3.3 (a) Far-field auto-spectra, (b) compensated, (c) smoothed and (d) as a function of Strouhal number. ................................. 27
3.4 Far-field sound pressure level in decibels (ref. 20 \( \mu \text{Pa} \)), at 75 jet diameters from the nozzle exit. ................................. 28
3.5 Compensated near-field auto spectra (a) for all azimuthal positions \( \theta \) at \( x/D=7 \) and (b) as a function of Strouhal number; (c) for all downstream positions at \( \theta = 90^\circ \) and (d) as a function of Strouhal number. ............................................. 29
3.6 Normalized cross-correlation between the near-field pressure sensors as a function of angular separation for all downstream positions. . 32
3.7 Normalized cross-correlation between the near-field pressure sensor at \( \theta = 96^\circ \) for all downstream positions and each of the far-field microphones from \( \phi = 90^\circ \) (mic 1) to \( \phi = 15^\circ \) (mic 6). ................................. 34
3.8 Normalized cross-correlation between the near-field pressure sensor at \( \theta = 96^\circ \) for all downstream positions and the far-field microphone at \( \phi = 15^\circ \) (mic 6). ............................................. 35
3.9 Phase-angle of the cross-spectrum between near-field pressure sensor at \( \theta = 72^\circ \) and far-field microphone at (top) \( \phi = 60^\circ \) and (bottom) \( \phi = 15^\circ \). ............................................. 37
3.10 Maximum normalized cross-correlation between the near-field and far-field pressure as a function of downstream position, for all microphones mic 1 (\( \phi = 90^\circ \)) to mic 6 (\( \phi = 15^\circ \)). ......................... 39
3.11 Normalized cross-correlation between all near-field pressure sensors and each of the far-field microphones from \( \phi = 90^\circ \) (mic 1) to \( \phi = 15^\circ \) (mic 6), at \( x/D=7 \). ................................. 40
3.12 Normalized cross-correlation between all near-field pressure sensors and the far-field microphone at \( \phi = 15^\circ \) (mic 6), at \( x/D=7 \). 41
3.13 Energy contained in all azimuthal Fourier modes (0 to 7) for downstream positions \( x/D=1 \) to \( x/D=5 \). 43
3.14 Energy contained in all azimuthal Fourier modes (0 to 7) for downstream positions \( x/D=1.5 \) to \( x/D=11 \). 44
3.15 Comparison of the level of energy in azimuthal Fourier modes 0 to 4 as a function of downstream position. 45
3.16 Time series of the near-field pressure compared to (top) the mode 0 part of the pressure only, (middle) the mode 1 part of the pressure only and (bottom) the sum of modes 0 and 1. 46
3.17 Comparison of the normalized cross-correlation between the far field sound at \( \phi = 30^\circ \) and the mode-filtered near-field pressure at \( \theta = 96^\circ \), \( x/D=8 \). 47
3.18 Maximum of the normalized cross-correlation between the far field sound at \( \phi = 30^\circ \) and the mode-filtered near-field pressure, as a function of downstream position. 48
4.1 (a) Front view and (b) top view of the DT-PIV experimental setup. 60
4.2 Dual-time PIV timing schematic. 62
4.3 Fields of view of all four PIV cameras with calibration plate centered on the jet centerline. 65
4.4 RMS of the velocity change \( \Delta u \) as a function of \( \Delta t \), using LDA measurements, at several radial locations, \( x/D=6 \). 66
4.5 Instantaneous 3-component velocity fields measured by both systems with a delay \( \Delta t = 25 \mu s \). 68
4.6 Relative convergence error \( \xi \) as defined in the text for the axial and radial velocity components, at several radial locations, \( x/D=8 \). 71
4.7 (top) Typical \( U_x(r) \) average jet velocity profile and (bottom) exaggerated non-orthogonality of the PIV measurement plane with respect to the jet axis. 73
4.8 Mean cross-flow \( (U_r, U_\theta) \), showing a greater horizontal than vertical component of velocity, \( x/D=8 \). 75
4.9 Average absolute axial velocity difference between the measurements of PIV 1 and PIV 2 with (a) \( \Delta t = 2 \mu s \) and (b) \( \Delta t = 25.3 \mu s \), \( x/D=8 \). 76
4.10 Contours of (a) \( (U_{PIV2} - U_{PIV1}) \) and (b) \( (\sigma_{U_{PIV2}} - \sigma_{U_{PIV1}}) \), in m/s, \( x/D=8 \). 77
4.11 Mean axial velocity profiles. 78
4.12 Mean axial velocity contours. 79
4.13 Mean cross-flow \( (U_r, U_\theta) \) at (top) \( x/D=3 \) and (bottom) \( x/D=4 \). 80
4.14 Mean cross-flow \( (U_r, U_\theta) \) at (top) \( x/D=5 \) and (bottom) \( x/D=6 \). 81
4.15 Mean cross-flow \( (U_r, U_\theta) \) at (top) \( x/D=7 \) and (bottom) \( x/D=8 \). 82
4.16 Mean cross-flow \( (U_r, U_\theta) \) at (top) \( x/D=9 \) and (bottom) \( x/D=10 \). 83
4.17 Turbulence intensity for velocity components \( u \) (top), \( u_r \) (middle) and \( u_\theta \) (bottom). 85
4.18 Cumulative and individual convergence of the POD eigenvalues. 97
4.19 Individual energy content of the POD eigenvalues (a) from \( n=1 \) to \( n=150 \) and (b) from \( n=1 \) to \( n=25 \). 98
4.20 Eigenfunctions of the axial velocity component \( \phi_u^{(n)} \), modes \( n=1 \) to \( n=6 \). ........................................... 100

4.21 Eigenfunctions of the axial velocity component \( \phi_u^{(n)} \), modes \( n=7 \) to \( n=12 \). ........................................... 101

4.22 Eigenfunctions of the axial velocity component \( \phi_u^{(n)} \), modes \( n=14, 15, 16, 18, 19 \) and \( 20 \). ........................................... 102

4.23 Eigenfunctions of the axial velocity component \( \phi_u^{(n)} \), modes \( n=13, 17, 21, 22, 23 \) and \( 24 \). ........................................... 103

4.24 Eigenfunctions of the axial velocity component, (left) \( \phi_u^{(1)} \) and (right) \( (\phi_u^{(2)} + \phi_u^{(3)} + \phi_u^{(4)})/3 \). ........................................... 104

4.25 Eigenfunctions of the axial velocity component, (left) \( \phi_u^{(10)} \) and (right) \( (\phi_u^{(9)} + \phi_u^{(11)} + \phi_u^{(12)})/3 \). ........................................... 104

4.26 Eigenfunctions of the axial velocity component, (left) \( \phi_u^{(21)} \) and (right) \( \phi_u^{(21)} \) with identification (red circles) of inner/outer Fourier mode-like structures. ........................................... 105

4.27 Axial fluctuating velocity component of (left) the original instantaneous PIV velocity field and (right) the 100 POD mode reconstruction, at three independent times (\( t=236, 238 \) and \( 249 \)), at \( x/D=8 \), color scale from -60 to 60 m/s. ........................................... 106

4.28 Identical plots as in Figure 4.27 with the mean axial velocity field added, color scale from 0 to +190 m/s. ........................................... 107

4.29 In-plane fluctuating velocity components of (left) the original instantaneous PIV velocity field and (right) the 100 POD mode reconstruction, at three independent times (\( t=236, 238 \) and \( 249 \)), at \( x/D=8 \). ........................................... 108

4.30 (a) Axial fluctuating velocity component of the original instantaneous PIV velocity field, and POD reconstructions using (b) 12 modes (30\% of TKE), (c) 34 modes (50\% of TKE), (d) 88 modes (70\% of TKE), (e) 305 modes (90\% of TKE), and (f) 4000 modes (100\% of TKE), time \( t=454 \), \( x/D=8 \), color scale from -60 to +60 m/s.109

4.31 Time series of the 12 predicted POD modes of LODS 1, \( x/D=8 \). ........................................... 124

4.32 Time series of the 12 predicted POD modes of LODS 2, \( x/D=8 \). ........................................... 125

4.33 (top) Time series of the predicted centerline fluctuating axial velocity and (bottom) norm of the Morlet wavelet transform of the time series, \( x/D=8 \). ........................................... 127

4.34 Spectra of the centerline axial velocity component, \( M=0.6, x/D=1 \) to 6, from Tinney (2005). ........................................... 128

4.35 Time series of the 12 first predicted POD modes of LODS 3, \( N_{LODS}=30 \), \( x/D=8 \). ........................................... 129

4.36 LODS 3, (top) Time series of the predicted centerline fluctuating axial velocity and (bottom) norm of the Morlet wavelet transform of the time series, \( N_{LODS}=30 \), \( x/D=8 \). ........................................... 130

4.37 Time series of the 12 first predicted POD modes of LODS 4, \( N_{LODS}=48 \), \( x/D=8 \). ........................................... 131

4.38 LODS 4, (top) Time series of the predicted centerline fluctuating axial velocity and (bottom) norm of the Morlet wavelet transform of the time series, \( N_{LODS}=48 \), \( x/D=8 \). ........................................... 132
4.39 Time series of the 12 first predicted POD modes of LODS 5, \( N_{LODS} = 65, x/D = 8 \). ........................................ 133

4.40 Time series of the 12 first predicted POD modes of LODS 6, \( N_{LODS} = 65, x/D = 8 \). ........................................ 134

4.41 LODS 6, (top) Time series of the predicted centerline fluctuating axial velocity and (bottom) norm of the Morlet wavelet transform of the time series, \( N_{LODS} = 65, x/D = 8 \). ........................................ 135

4.42 Phase representation of \( a_2 \) as a function of \( a_1 \), \( x/D = 8 \). ........................................ 136

4.43 Phase representation of \( a_2 \) as a function of \( a_1 \), \( x/D = 8 \). ........................................ 136

4.44 LODS 6, \( N_{LODS} = 65 \), phase representation of (a) \( a_2 = f(a_1) \), (b) \( a_{10} = f(a_9) \) and (c) \( a_{26} = f(a_{25}) \), \( x/D = 8 \). ........................................ 137

5.1 Schematic of cavity/diaphragm system with exit slot. ................. 147

5.2 Front and side views of the synthetic jet actuating system. ............. 148

5.3 View of the actuating system installed on the jet nozzle with acoustic shield. .................................................. 150

6.1 Plots of the phase angle of the cross-spectra between the near- and far-field pressure, far-field microphones at \( \phi = 15^\circ, 60^\circ \) and \( 90^\circ \) (left to right), \( x/D = 1, 2, 3, 4 \) and \( 5 \) (top to bottom). ......................... 165

6.2 Plots of the phase angle of the cross-spectra between the near- and far-field pressure, far-field microphones at \( \phi = 15^\circ, 60^\circ \) and \( 90^\circ \) (left to right), \( x/D = 6, 7, 8, 9 \) and \( 10 \) (top to bottom). ......................... 166
Nomenclature

Coordinate systems

\(x, y, z\) axial, vertical and horizontal directions (Cartesian)
\(x, r, \theta\) axial, radial and azimuthal directions (polar)

Roman symbols

\(a_n(t)\) \(n^{th}\) POD mode time dependant expansion coefficient
\(B_{nn}\) Fourier azimuthal spectrum of the near field pressure
\(c_o\) speed of sound in air at ambient temperature
\(C\) snapshot POD kernel
\(dt\) time delay between laser Q-switch openings
\(dx, dy\) spatial resolution of PIV measurement
\(D\) jet diameter
\(\mathcal{D}\) spatial integration domain
\(D_n, L_{nj}\) constant and linear coefficients of the LODS
\(Q_{njk}, C_{njkl}\) quadratic and cubic coefficients of the LODS
\(f_s\) sampling frequecy
\(G_{nf}\) single-sided cross-spectrum between the near- and far-field pressure
\(m\) azimuthal Fourier mode
\(M\) Mach number
\(n_c\) number of velocity components
\(n_s\) number of samples per block
\(N\) total number of POD modes
\(N_{grid}\) number of grid points in the measurement area
\(N_{LODS}\) number of modes used in the LODS
\(N_t\) total number of PIV snapshots
\(p_f\) far-field pressure
\(p_n\) near-field pressure
\(R_{ij}\) classical POD kernel
\(R_{nf}\) cross-correlation function between the near- and far-field pressure
\(Re\) Reynolds number
\(S_{nf}\) double-sided cross-spectrum between the near- and far-field pressure
\(St\) Strouhal number, \(St = f.D/U_c\)
\(T_{ij}\) Lighthill stress tensor
\(U_c\) jet centerline velocity
\(\tilde{u}\) velocity field due to the actuation
\(\tilde{u}, \tilde{v}, \tilde{w}\) axial, vertical and horizontal components of the instantaneous velocity
\(u, v, w\) axial, vertical and horizontal components of the fluctuating velocity
\(U, V, W\) axial, vertical and horizontal components of the mean velocity
\(u_x, u_r, u_\theta\) axial, radial and azimuthal components of the fluctuating velocity
\(U_x, U_r, U_\theta\) axial, radial and azimuthal components of the mean velocity
Greek symbols

δ_{ij}  Kronecker’s delta function
Δθ  azimuthal separation
Δt  time delay between the two PIV systems
ε  total TKE
λ^{(n)}  n^{th} mode POD eigenvalue
Λ^{(n)}  n^{th} mode POD eigenvalue overall contribution to TKE (in %)
ν  kinematic viscosity
ϕ  polar angle of far-field measurement, from jet centerline
Φ^{(n)}(x, y)  n^{th} POD mode spatial eigenfunction
Ψ  misalignment angle
ρ  density of air
ρ_{nf}  normalized cross-correlation function between the near- and far-field pressure
σ_u  standard deviation of the axial velocity
τ_{ij}  viscous stresses
θ  azimuthal angle centered on the jet centerline
ξ  relative convergence error
ζ  time dependent forcing amplitude

Acronyms

A/D  analog to digital conversion
CAD  computer aided design
CFD  computational fluid dynamics
DNS  direct numerical simulation
DT-PIV  dual-time PIV
IFT  inverse Fourier transform
LDA  laser doppler anemometry
LES  large eddy simulation
LODS  low-order dynamical system
LSE  linear stochastic estimation
ODE  ordinary differential equation
PDE  partial differential equation
PIA  particle image accelerometry
PIV  particle image velocimetry
POD  proper orthogonal decomposition
ROM  reduced order model
RMS  root-mean square
SPL  sound pressure level
SVD  singular value decomposition
TKE  turbulent kinetic energy
TR-PIV  time-resolved PIV
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Syracuse, New York
August 8th, 2007

Jeremy Pinier
To my family.
Introduction

Aerodynamic jet noise is a problem that has attracted much attention since the middle of the twentieth century, ever since the appearance of “man-made” high-speed flows that happen to generate a great amount of sound as a counterpart of the quantity of interest which is in most applications thrust. The interest first came from the fundamental motivation of trying to explain the physical mechanisms that are the source of jet noise, trying to describe what aerodynamic events in the high-speed flow are most efficient at radiating highly intense acoustic pressure fluctuations that translate in the far-field to broadband noise. This noise, highly irritating to the human ear, then started gaining more attention as commercial air transportation became common and the comfort of humans around airports and in airplanes started being a real concern. Today, the actual control of jet noise is at its early stages and many planes now fly with engines that were designed with passive flow control devices commonly known as chevrons or tabs that enhance mixing between the high-momentum high-temperature core jet flow and the lower-speed cold secondary jet flow. These devices have reduced the noise radiated by these high-speed jet exhausts without compromising the thrust efficiency. It is important to note that every flow control strategy needs to be thought of as a performance enhancer and the mere fact that the overall efficiency of the process is negative makes the flow control irrelevant. Commercial jet engines designed and built today not only need to comply with the current federal regulations concerning noise standards but also the ones that will be in place at the end of the life of the engine, which could be twenty years after it is built. With
international regulations becoming more and more demanding on this issue, jet noise has become an ever more challenging problem. It is also gaining much attention in the air force/navy military applications where the whole range of aircraft, from unmanned air vehicles (UAV) to fighter aircraft need to be more and more stealthy and discrete. Where radar signature is now minimized on the most recent aircraft, their noise signature is far from being a solved problem. Flow control at the first stages of the design process is needed for such complex problems where so many disciplines come into play, from propulsion to thermodynamics, turbulence, acoustics and control theory. Active flow control techniques for jet noise reduction have not yet provided enough consistency and cost-efficiency to be found in industrial applications but represent the solution with the most potential for furthering and optimizing the jet noise reduction capability provided by passive techniques. Both active and passive flow control could be complementary in achieving our so challenging goal.

A problem that is specific to the laboratory and that makes all scaled-down experiments more challenging than the full-scale problem is, is that all frequencies associated with the former are significantly higher than those tied to the latter. By consequence, it is for instance much more challenging to find actuating systems with high frequency responses for laboratory use, a problem that might not pertain when applied to the real full-scale application. In the same manner, frequencies we are concerned about are found to be tied to a certain region in the jet flow. These frequencies being shifted down when moving from the scaled-down to the full-scale problem, the regions of concern will also shift in space which can make the laboratory experiment somewhat misleading. With the very fast-paced progress in the development of “smart” materials and piezoelectric components, high frequency actuating devices are now easier to find “off-the-shelf” at a reasonable cost for laboratory experiments. Piezo-electric type actuators have been transferred to full scale flight tests [95] and showed very satisfactory results in
terms of performance enhancement, reliability and practicality. These types of actuators have been chosen in the present work as a simple and low-cost test-bed with enough dynamic response for a first closed-loop flow control attempt for jet noise reduction. Active control techniques will certainly benefit in the close future from the recent developments in the field of micro-electro-mechanical systems (MEMS) that integrate sensors, actuators, mechanical elements and electronics on a silicon chip. The small size, reliability and efficiency of such devices are very attractive for aerospace applications, where minimizing the size and weight of additional systems such as sensors and actuators is fundamental. It doesn’t seem unrealistic that aircraft using active flow control will transfer to full scale flight tests within the next decade, fusing information from multiple sensors in real-time to make decisions on actuator output and enhance the performance and efficiency of flight.

The core of this thesis is focused on experimentally measuring and building a high quality database of the acceleration field of the high-speed compressible jet in the “loudest” region of the flow and building from this database a low-order dynamical system (LODS) of the jet dynamics at one downstream position (x/D=8) to gain prediction capabilities to be used in future flow control applications on the jet. There have been several successful attempts made at measuring either the time derivative or the spatial derivative of a velocity field in a whole plane using multiple particle image velocimetry (PIV) systems since the year 2000 and this type of experimental measurement has been made possible thanks to the quality and reliability of the advanced optical/laser instrumentation that is available today for fluid dynamics measurements.

Before describing in detail the main experiment and exploiting its results, the details of a preliminary experiment will be shown where a tentative at understanding the link between the acoustics-dominated far pressure field and the
hydrodynamics-dominated near pressure field is carried out. Through this preliminary experiment, interesting and quite original results were found that bring new information on how the sound propagates to the far-field and gives insight as to what kind of flow control strategy could be adopted to minimize the sound propagation. The core experiment is then presented with extensive details on the experimental setup and calibration of the systems. The results are then discussed in the context of their potential use for elaborate flow control algorithms that use predictive estimates of the flow state. The low-dimensional tools used to perform this task are also presented thoroughly since they are an integral part of the process of building a low-order model of the flow. A third part concentrates on using the information gathered through the first two experiments toward the design of an actuator “glove” that has the potential of providing the slight modifications to the near-field pressure to minimize sound propagation. The actuating system is designed for future implementation of both open- and closed-loop control strategies, using the flexibility and dynamic range of the chosen actuators to explore the effects of all the degree-of-freedom of the actuators on the far-field sound.
Chapter 1

Background on jet noise identification and control

1.1 Towards understanding jet noise

Jet noise studies along with the study of most aeroacoustics problems started in the early 1950’s with the theoretical work of Lighthill [66] [67] and his acoustic analogy theory. The original idea was to rearrange the compressible equations of fluid motion into the form of an inhomogeneous wave equation with the right-hand side terms thus representing the wave (or sound) sources.

\[ \frac{\partial \rho}{\partial t} - c_o^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \]  \hspace{1cm} (1.1)

where \( \rho \) is the density of air, \( c_o \) is the speed of sound at ambient temperature and \( T_{ij} \) is the Lighthill stress tensor defined as:

\[ T_{ij} = \rho u_i u_j + (p - \rho a_{\infty}^2) \delta_{ij} - \tau_{ij} \]  \hspace{1cm} (1.2)

with \( \delta_{ij} \) the Kronecker delta function and \( \tau_{ij} \) the viscous stress tensor.

From this pioneering theoretical work grew much interest in the field of aeroacoustics to try to identify sound sources in high-speed flows and understand the physical mechanisms at their origin. A dimensional analysis applied to the solution to Lighthill’s wave equation led to the establishment that the acoustic power
radiated by a high-speed jet must vary proportionally to $U^8$. This dependence of the noise on velocity has been since verified experimentally and explains the importance of the study of the aeroacoustics of high-speed flows. There has been major progress towards understanding how sound propagates to the far-field since Lighthill’s work on jet noise. The highly convective nature of the jet flow results in a much higher noise radiation in the direction of propagation of the sound sources inside the flow, which is a finding that will be again verified in the present work.

A major step in understanding noise sources in the axisymmetric jet was triggered by Mollo-Christensen’s experimental work [74] along with Crow and Champagne’s studies [34] that changed the way turbulent structures were interpreted in high Reynolds number flows. Indeed, it was thought that highly turbulent flows were made up of a random distribution of small eddies, each one of them emitting sound and convecting downstream interacting with the other eddies. The work of Mollo-Christensen for the first time identified orderly (or more regular) structures in the near-field pressure of the jet flow indicating that there might be larger scale dynamics involved in the noise production mechanisms. The flow visualizations of Crow and Champagne showed the presence of highly convective coherent structures. This was also followed by experimental measurements of the near-field pressure by Fuchs [41], who found large correlation lengths both azimuthally and in the streamwise direction around the jet from the nozzle exit down to eight jet diameters from the exit after the collapse of the potential core. He interpreted these results by describing the presence of a wave-like structure that is convected downstream and that interacts with the azimuthally coherent structure. Michalke and Fuchs [73] then theoretically found that the lower order azimuthal components of the near-field pressure are the most efficient radiators of sound. This result is partially verified in the present work, having found that mode 0 (the axisymmetric mode) is the only most efficient radiator of sound. This result will be discussed in Chapter 3. It is now widely believed in the aeroacoustics community that there are two main sources of noise in the jet flow, one originating from the fine-scales of
turbulence and dominating the noise sensed perpendicularly from the jet axis and at aft angles, and one coming from large-scale structures azimuthally coherent and radiating mostly in the direction of the jet flow [99]. In recent years however, the efficiency of the Lighthill source term at representing the actual sources that radiate sound to the far-field has been slightly rethought and the problem has been recast into examining term by term the Lighthill stress tensor to identify which part of it (compressibility or incompressibility driven terms) is actually important for modeling the noise sources [22]. There have also been technological improvements on the experimental side and measuring unsteady density fluctuations in high-speed flows (which is crucial to the jet noise problem) has recently been made possible. Using a molecular Rayleigh scattering technique, Panda et al. [83] were recently able to measure simultaneously the density and velocity fluctuations at one point in a time-resolved manner. Along with the simultaneous measurement of far-field sound and through cross-correlations they were able to show that the quantity that best correlated with the far-field sound was $\rho uu$ ($u$ being the streamwise component of velocity), whereas $\rho vv$ was found to correlate rather poorly with the far-field sound, which seems to indicate once more that certain quantities measured directly in the flow are tightly linked to the sound radiated, even though the acoustic wave undergoes significant refraction as it propagates through the high-speed hydrodynamic shear-layer. There has been in recent years an important amount of experimental work that has shed light on the mechanisms relating the jet flow, its surrounding pressure field and the far-field perceived sound. Multi-point and single-point measurement techniques have improved greatly since the introduction of laser-based techniques such as laser doppler anemometry (LDA), particle image velocimetry (PIV) and Rayleigh scattering. The non-intrusiveness of these techniques is what made in-flow aeroacoustic measurements possible.

Quite surprisingly, subsonic jet noise has shown significantly more challenges to the understanding of its radiated noise as compared to supersonic jets. The
latter have been studied for a little less time than subsonic jets but more is known about the sound sources in these than in their subsonic counterparts. The main reason for this is that noise can be related to the Mach wave radiation process which is very efficient at generating noise. This topic is addressed by Tam [97] and the reader is referred to his review of the progress made towards jet noise since 1952 for a more thorough history of jet noise.

The increase in computational capabilities within the last decade has made possible realistic simulations of the aeroacoustics of high-speed jets. Even though the Reynolds numbers are usually low compared to realistic ones, the Mach numbers of simulations are now comparable to experiments and much has already been gained from these studies. To only cite a few, using large eddy simulation (LES), Bailly and Boguet [15] have been able to accurately predict the acoustics and flow fields of a M=0.9 circular jet at a Reynolds number of 65,000. Wei and Freund [109] have proved, through direct numerical simulation (DNS) that a Mach 0.9 jet could be made much more quiet using adjoint methods of flow control, which is only feasible in simulations but nonetheless can guide flow control strategies by understanding the physical differences between a loud and a quiet jet.

1.2 The dynamics of the axisymmetric jet

The motivations behind studying the dynamics of the axisymmetric jet are obviously very closely tied to the problem of jet noise. Acoustic measurements in the near and far field of a flow being much more easy to implement than direct measurements of in-flow quantities, the earlier studies were mostly concentrated on investigating the relationship between near and far field pressure [41] [73] and debating upon the presence of organized large-scale structures in high Reynolds number flows [39]. The problem of jet noise becoming more challenging than pre-
viously thought, measurements needed to be performed inside the jet flow and insight into the actual mechanisms driven by the axisymmetric nature of this particular flow was needed. This area became an area of its own and the study of the dynamics of axisymmetric jets is still today an area where discoveries still remain to be made before we can fully understand the dynamics of this complex flow.

The extensive experimental studies that have been led were made possible in great part by improvements in the accuracy and practicality of measurement techniques. The original motivation in the seventies came from the evidence of the presence of large scale organized structures in turbulent flows.

Figure 1.1: Laser-based visualization of the developing instabilities in the axisymmetric jet flow, Re=10,000.

Figure 1.1 shows a visualization performed by the author on a low speed (3 m/s) axisymmetric jet flow that clearly shows how the initial instabilities in the shear layer develop into large scale structures in the downstream turbulent flow. Through flow visualization, Brown and Roshko [20] were able to show clear images of large-
scale structures triggered by a Kelvin-Helmholtz type instability at the edge of a mixing layer. Even when increasing the Reynolds number, the large-scale structures remain unchanged but smaller-scales are produced. Interesting interaction of these structures was also observed with pairing of large vortices as a higher momentum vortex overtakes a slowing vortex ahead. These types of mechanisms are the kinds that are sought to be understood in the jet flow, where axisymmetry makes these processes more difficult to capture and where it is believed that particular mechanisms specific to the axisymmetric nature of the flow exist. The work of Glauser et al. [46] [44] has certainly shed light on what might be a model for a mechanism of vortex interaction and breakdown in the jet.

Figure 1.2: The four stages of turbulence production (left to right), model proposed by Glauser et al. [46] showing the break-up of a large coherent structure into higher order azimuthal structure due to the passage of an inner ring through the outer low momentum ring.

Figure 1.2 shows a sketch of the model, as proposed by Glauser et al. [46] that would explain how an azimuthally coherent vortex ring (mode 0 dominant) can be overcome by a smaller high momentum vortex ring with the result of breaking the former into a higher order structure (mode 5, 6 or 7), this event being
often referred to as the “volcano” event. Further experimental evidence of the
existence of this event in the jet was shown by Citriniti [28] and Tinney [102].
This type of mechanism could be a candidate responsible for far-field noise since
it is a high-strain, short-time event that occurs in the region of collapse of the
potential core, downstream of which most of the sound is produced in jets. A
significant amount of data has been collected since at many different Reynolds
numbers and using many different types of measurement tools. To cite only a few
of the most significant databases that have contributed to understanding the jet
flow, Arndt et al. showed how hydrodynamic fluctuations behaved differently than
acoustic fluctuations in a spectral sense as they are radiated. In the overlapping
region, close to the outer edge of the shear layer, both fields (hydrodynamic and
acoustic) are very much present but their spectral content as a function of the
product of wave-number times radial distance ($kr$) is clearly different. Indeed the
hydrodynamic fluctuations seem to decay as $(kr)^{-6.67}$ (usually visible for $kr < 2$)
and the acoustic fluctuations are characterized by a slope of $(kr)^{-2}$ (usually visible
for $kr > 2$). This indicates a practical way to separate within the measurement
the radiating acoustic pressure waves from the hydrodynamic pressure waves con-
vecting with the mean flow.

1.3 Jet noise control

Jet noise control has reached a plateau in recent years since the use of passive
devices such as chevrons that have shown to successfully reduce perceived far-field
sound by several decibels. This type of control is the only technology that has
transferred to commercial jet engines and is the result of many years of research
on that subject. The challenge before us now lies in using active control, in con-
junction with passive techniques, to manipulate fluid flows in a smarter and more
tailored way to maximize the control impact on the flow at the same time as
minimize the control output needed. A brief review of the existing literature in
the domain of jet flow control follows, separating passive from active control, and
within active control, open- from closed-loop control. The present work lays the
grounds for closed-loop control in the high-speed jet, it will therefore be put into
context within the different efforts that have been led towards active flow control
of jets for noise reduction.

1.3.1 Passive techniques
The first radical improvement in jet engines for noise reduction since the first one
was built in the forties has been the turbofan, which consists in increasing the mass
flow through the engine but lowering the exhaust air velocity by separating the air
passing through the engine into a high pressure core stream and a low pressure co-
flowing stream. This resulted in a much quieter jet plume since the velocities were
significantly lowered. The more recent high bypass ratio turbofan (as compared
to the low bypass turbofan) found on today’s large size civil jetliners have reached
levels of fuel efficiency, specific thrust and noise emission that are so competitive
that it constitutes a constant challenge to improve them. However international
regulations concerning jet noise in particular are becoming very restrictive. In-
deed, a group of representatives of the European aviation industry published a
document in 2001 (“European aeronautics: A vision for 2020” [5]) with the vision
and goal that, by 2020, airplanes will have to: 1) Reduce fuel consumption by 50%,
2) Reduce perceived external noise by 50%, 3) Reduce nitrogen-oxide by 80%, and
4) make significant efforts to minimize the impact on the environment of the manu-
facture and disposal of aircraft and related products. These very challenging goals
have pushed engine manufacturers to look at alternative ideas to reduce noise and
the use of chevrons was a simple and effective way to get started along the path of
jet flow control. Nowadays, chevrons (also known as tabs) along with lobe mixers
are becoming a common sight on commercial airplanes. By increasing mixing of
the hot core flow with the lower velocity and temperature bypass flow, the shear is not as brutal between the two flows and the emitted noise is consequently lessened. The presence of the chevrons creates longitudinal vortices that force mixing without compromising thrust. It is however still unclear how the chevron geometry and spatial distribution can be optimized, if there even exists an optimal spacing and geometry. There has been much research in trying to understand and quantitatively measure the impact of the chevrons on the flow. Among others, Bridges et al. [19] have performed a parametric study on the use of different configurations of chevrons on high-bypass ratio and single stream nozzle arrangements. They have found that by disrupting the large-scale coherent structures, the frequency content of the far-field noise is shifted towards higher frequencies. If one has the desire to impede frequencies on a broader range, a closed-loop control (or at least active control) strategy would have much more potential since it would be possible to sense and actuate accordingly. This topic is discussed in the following section.

1.3.2 Active techniques

Active techniques imply that energy is added to the flow under the form of constant mass injection, oscillatory excitation, electrical input, etc. There is therefore a power consumption associated with all active flow control techniques which needs to be addressed; as there is also a penalty to having a permanent geometrical addition to the system in the case of passive control (added mass, increased drag...). Active flow control techniques are divided into open-loop and closed-loop control approaches. In open-loop control, the actuator input is stationary in time and its characteristics are chosen by the operator beforehand. It is therefore very closely related to passive techniques in that the improvement to the flow in some targeted situation will be a penalty in another. It is therefore not optimized in terms of energy consumption and efficiency but much can be learned from an initial open-loop investigation of the impact of a given actuation technology on the flow. In
the closed-loop case, sensors are used to adjust the actuation input in real-time depending on the flow dynamics and a model of the flow can be used to predict its evolution and optimize the prescribed actuation. It is therefore much more cost efficient and has a greater potential for the control of highly unsteady flows where the control input needs to be continuously adjusted to follow the dynamics of the flow. The present investigation brings knowledge that can guide ways to actively control the jet shear layer to mitigate the far-field perceived sound and an active flow control device is designed in this aim.

**Open-loop control**

The initial motivation for controlling the axisymmetric jet was the stability analysis of the preferred modes in the jet. Forcing particular frequencies on the initial growth of the shear layer could regularize the flow and a better understanding of the underlying physics was yielded. Acoustic excitation of jet instabilities was performed, among others, by Ginevsky et al. [43] and also Cohen and Wygnansky [29] who showed that not only modes 0 and 1 (as shown previously) but also higher modes were unstable in the initial shear layer development. The interest in recent years has been guided towards manipulating the jet flow to enhance mixing and more recently to reduce the far-field sound in high-speed jet applications. Demonstrations of different active flow control systems have all shown the potential of such techniques to improve slightly on passive techniques in that the targeted range of frequencies to be controlled can be actively chosen. However it has been shown that the decrease in intensity of the targeted range almost systematically provokes an increased intensity in another range of frequencies. Fluidic injectors have been used to replace chevrons and act to create streamwise vorticity. This kind of actuation was applied by Alkislar et al. [2] on a Mach 0.9 jet, Laurendeau [63] on a Mach 0.3 jet and showed slight benefits compared to physical chevrons on the far-field sound reduction. Samimy et al. [94] have studied the receptivity of the jet shear layer to plasma actuation. This type of actuation is very attractive since
it benefits from a very large frequency bandwidth (up to 200 kHz) and its power consumption has shown to be on the order of 30 W per actuator which is reasonably small. Control on the jet was shown to be very effective at forcing the dynamics at particular actuation frequencies and the far-field sound was greatly increased when forcing at the preferred mode (St=0.2-0.5). Only a slight decrease in noise was found (0.5 to 1 dB) when forcing within a Strouhal range of 1.5 to 3.5 but this technique shows great potential for use in an optimized closed-loop architecture due to its high frequency response. Synthetic jet actuation has also been successfully used by Tamburello and Amitay [100] to manipulate jets but the focus was guided towards jet vectoring. This technique also presents the advantage of having a relatively high frequency response and it presents the additional convenience of being relatively simple to implement.

**Closed-loop control**

Closed-loop control remains a very scarcely used technique in high-speed jet flow control since the lack of high frequency actuating devices has hindered such endeavors. Annaswamy *et al.* [4] have applied closed-loop flow control strategy based on the POD to reduce the intensity of edge-tones on a Mach 1.5 impinging jet but the active manipulation of the flow was made on much longer time scales than that of the flow dynamics. In low Reynolds number flows (mostly laminar) however, closed-loop control has been widely applied [30] [82] [26] especially computationally where advanced methods such as adjoint methods can be implemented. It has shown to be very beneficial when the dynamics of the flow are clearly measurable (Karman vortex shedding, periodic cavity resonance...) but its implementation in fully turbulent flows has required the use of low-dimensional methods to extract these large scale features [88] to be used to guide the control input. The use of closed-loop methods in high-speed flows seems today to have the most potential in improving the current state-of-the-art in flow control and aeroacoustic manipulation of high-speed jets.
Chapter 2

The Syracuse University large anechoic chamber and jet facility

2.1 Skytop anechoic chamber

Acoustical measurements can be of different nature. When one is seeking only to quantify the total acoustic power radiated by an object, a reverberation room is usually used where a diffuse sound field is established. When one is seeking to extract directivity information, or trying to specifically localize sound sources within the system, a free-field measurement environment needs to be established where total power and power spectral quantities can be computed from the directional data. In this case, measurements need to be performed in an anechoic chamber with acoustically treated walls, so as to absorb all (or most of) the incident acoustic pressure waves. The Syracuse University large anechoic chamber (206 m$^3$) located on the South Campus of Syracuse University was built in the 1970’s by Dr. Dosanjh in order to perform large scale flow control experiments on supersonic jets (both single-stream and coaxial) [35]. Previous experiments were carried out in a smaller anechoic chamber at Syracuse University which had a size of 3.2 m $\times$ 4 m $\times$ 2.8 m (wedge-tip to wedge-tip) [75]. Impinging jet flow control research was performed in this facility on a supersonic jet to study the impact of such flow control on the far-field noise. The present large anechoic chamber has a wedge-tip to wedge-tip size of 8 m $\times$ 6 m $\times$ 4.3 m, which enables large scale experiments
for more accurate far-field measurements. The facility was recently refurbished by Tinney [102] and a new jet nozzle with flow straightening and screening was designed and installed along with remotely controlled valves and automatic control of the flow conditions. The design and construction of this facility is described in detail in Tinney et al. [104].

2.2 High-speed jet facility

2.2.1 Facility and flow conditions

A 2 stage Joy compressor discharges compressed air to five tanks with a total capacity of about 31 cubic meters (1100 ft$^3$) up to a pressure of 3.1 MPa (31 Bars or 450 psi). The compressor and tank room is located in a building adjacent to the anechoic chamber separating all noise and vibrations of the plant from the control and measurement room. An automatically controlled valve lets air from the tanks flow through over 200 ft of steel piping to reach the jet nozzle in the anechoic chamber. The axisymmetric jet nozzle exit is 50.8 mm in diameter with an area contraction ratio of 9:1, following a matched fifth order polynomial contraction. The capacity of the tanks allows for runs of 15 minutes at an exit Mach number of 0.85 at ambient temperature, 35 minutes at Mach 0.6 or a continuous run at Mach 0.3. A make-up air (MUA) unit brings outside air into a plenum chamber and pushes the air through a co-flow of about 3 m/s around the main high-speed jet flow. The MUA unit also enable accurate control of the ambient temperature in the chamber. Both the jet flow and the co-flow are exhausted out of the anechoic chamber through an acoustically treated eductor fan, hence balancing the pressure difference with the incoming air. Additionally a 407 kW Chromalox electric circulation heater was installed to be able to control the temperature of the jet exit from ambient temperature to about 400°C. The jet flow is controlled automatically and the pressure ratio (hence the velocity) is maintained constant at the nozzle exit.
thanks to a closed-loop automatic control. Figure 2.1 shows the anechoic chamber with the jet nozzle on the right side, the exhaust window on the far side with the eductor fan not visible in the back and the far-field microphone rake on the left side.

Figure 2.1: Syracuse University large anechoic chamber and high-speed jet facility, with 6 far field microphones (in red circles).

### 2.2.2 Measurements and data acquisition equipment

The laboratory is equipped with National Instruments PXI-based data acquisition hardware for simultaneous sampling of up to 24 channels. The three 8-channel A/D boards (PXI-4472) have 24 bits resolution and dedicated analog low-pass filters automatically set at half the sampling frequency to prevent any aliasing. Measurements carried out in the present work include:

- **Near-field pressure measurements**: 15 Kulite pressure transducers (model XCE-093-5G, 0-0.34 Bar) are used to measure pressure fluctuations near the outer edge of the axisymmetric shear layer. The diameter of the transducers is only 2.4 mm, which allows for a minimal intrusion in the partially hydrodynamic region of measurement. The Kulites require a 10 volt
DC excitation which is provided by five 3-channel Endevco model 136 DC differential-voltage amplifiers.

- **Far-field acoustics:** The jet’s far-field acoustics are measured along an array of 6 G.R.A.S. (type 40BE) 1/4 inch pre-polarized, free-field condenser microphones. Each individual microphone is positioned at 75 jet diameters from the center of the jet exit plane and placed at polar angles from the jet centerline axis (x) ranging from $\phi = 15^\circ$ to $\phi = 90^\circ$ with $\Delta \phi = 15^\circ$ increments. Figure 2.2 shows the layout of the anechoic chamber with the far-field pressure measurement locations.

- **Velocity and acceleration measurements:** Two Dantec Dynamics stereoscopic PIV systems are used to measure 3-component velocity fields and in-
stantaneous acceleration fields in cross-stream planes ranging from $x/D=3$ to $x/D=10$. Four CCD cameras ($1280 \times 1024$ pixels) and two double cavity, 532 nm wavelength, 200 mJ New Wave Research Gemini PIV lasers are mounted on a single base plate fixed on a motorized traverse so that the cameras and lasers stay always fixed with respect to one another throughout the runs at different downstream positions. This is important to ensure consistency of the calibration between each measurement location. The PIV images being streamed directly to external hard-drives, the frequency of the sampling is limited to 1 Hz. This however ensures perfect statistical independence of the samples. The averages over realizations independent in time can therefore be assimilated to ensemble averages. All the details pertaining to the dual-time PIV setup are found in Chapter 4.

2.3 Overview of the experiments

Two separate experiment are carried out in an attempt to further characterize the high-speed jet flow, understand its relationship to the acoustic sound field it radiates, and lay the foundations of a closed-loop control methodology for high-speed flows based on low-dimensional modeling of experimental data. The focus of the present work is noise reduction in the high-speed jet but the concepts can be generalized to many problems where experimental databases could be used to build models of flows.

2.3.1 Experiment 1: Near-field/far-field pressure measurements.

The first experiment is intended to further the understanding of the relationship between the near and far pressure fields of the high-speed jet flow. In this aim, measurements of the pressure close to the outer edge of the axisymmetric shear layer were performed simultaneously with measurements of the acoustics in the far-field of the Mach 0.85 jet (or 291 m/s at 20°C). This Mach number was chosen
to approach typical application values and avoid transonic effects. The near-field pressure rake is circular and consists of 15 transducers equally spaced around the jet flow. The far-field acoustics are measured with 6 microphones located at 75 jet exit diameters from the center of the nozzle exit plane and spaced equally from 90° to 15° from the jet centerline axis. The investigation uses cross-correlations between these signals to learn more about the noise sources’ location in the flow, the type of sound source and the propagative nature of the near-field pressure decomposed into azimuthal modes. Knowledge is gained towards designing actuating systems for closed-loop jet noise control. The detailed description of this experiment and the presentation of the original results are laid out in Chapter 3.

2.3.2 Experiment 2: Measurement of the Eulerian acceleration field.

In an effort to gain predictive capabilities and develop empirical low order dynamical systems based on experimental data, a direct measurement of the derivative of the velocity (i.e. the Eulerian acceleration) is needed. The use of two stereoscopic PIV systems in the same plane of measurement allows the sampling of a pair of successive velocity fields with a prescribed delay which is then used to compute the instantaneous acceleration. A Mach number of 0.6 (or 206 m/s at 20°C) was chosen to allow for more accuracy of the acceleration measurement. This type of data-set is quite unique in such a high speed flow since it requires a great amount of advanced equipment and presents challenges due to the high-speed nature of the flow that can compromise the accuracy of the measurement if not rigorously addressed. Chapter 4 describes the experimental procedure in detail and presents the low-dimensional methodology that enables empirical dynamical systems development. The method is successfully applied at one downstream position (x/D=8) and recommendations are made for the future development of a 3 dimensional dynamical system of the jet flow.
Chapter 3

Relationship between the near- and far-field pressure

3.1 Motivation

The interest in the first experiment is focused on finding the location of the most efficient sound producing events in the turbulent jet flow and also understanding and characterizing the sound source at the origin of the far-field noise. In a practical case, control on the jet flow would be produced at the lip of the nozzle. A knowledge of the relationship between the near-field pressure surrounding the jet flow - object potentially controlled - and the perceived far-field sound - control objective to be minimized - is fundamental as a guidance for the control strategy. Many theoretical (Michalke and Fuchs [73]), computational (Freund [40], Bogey & Bailly [15]) as well as experimental (Arndt et al.[6], Ukeiley and Ponton [107], Jordan et al. [58], Tinney et al. [103], Hall et al. [52], Barré et al. [11], Guitton et al. [50]) studies have been performed on the near-field pressure surrounding the axisymmetric jet to understand the relationship between the flow itself and its bordering pressure dynamics. All studies reveal the low-dimensional nature of the near pressure field around the jet flow, with dominating low-order azimuthal modes 0, 1 and 2. However, one of the most challenging objectives has been to accurately relate the near-field pressure surrounding the flow to the far-field acoustic pressure so as to estimate a perceived sound from the knowledge of the near-flow...
quantities only. In an effort to understand from an experimental point-of-view the nature of the near-field pressure around the jet and the propagation mechanisms to the far-field, simultaneous measurements of both the near-field pressure and the far-field sound are made in the anechoic chamber at Syracuse University on the Mach 0.85 jet flow at a Reynolds number of $9.8 \times 10^5$.

3.2 Experimental setup

3.2.1 Facility and flow conditions

The experiment is led in the 206 m$^3$ Syracuse University Skytop Anechoic chamber described in Chapter 2. The design and construction of this facility is detailed in Tinney et al. [104]. An acoustically treated eductor fan draws the jet air out of the chamber, which is equipped with a Make-Up Air (MUA) unit that balances the pressure deficit in the chamber due to the eductor fan and enables accurate control of the ambient temperature of the chamber. Additionally a 407 kW Chromalox electric circulation heater was installed to be able to control the temperature of the jet exit from ambient temperature to about 300$^\circ$C. The axisymmetric jet nozzle exit diameter is 50.8 mm and the flow was set at an exit centerline Mach number of 0.85 (or 291 m/s at 20$^\circ$C), to both approach typical application values and also avoid transonic effects. At this Mach number, the spreading of the outer shear layer was found to be 11$^\circ$ and the inner shear layer 5.5$^\circ$ with the collapse of the potential core at around 6 jet diameters downstream [104].

3.2.2 Measurements and data acquisition

Simultaneous sampling of the near-field and the far-field pressure is performed:

- The near-field pressure measurements are made along a circular ring array of 15 Kulite transducers (model XCE-093-5G, 0-0.34 Bar) evenly spaced in the azimuthal direction by $\Delta \theta = 24^\circ$. Figure 3.1 shows how the sensors are distributed
around the ring and gives the orientation of the angle $\theta$ with respect to the jet axis. The diameter of the transducers is only 2.4 mm, which allows for a minimal intrusion in the partially hydrodynamic region of measurement. The 15 Kulites require a 10V DC excitation which is provided by five 3-channel Endevco model 136 DC differential-voltage amplifiers. These amplifiers are signal conditioning units that have built in low-pass filters but these are not used in the present experiment since the National Instruments data acquisition boards are equipped with dedicated low-pass filters that automatically set the cutoff frequency at half of the sampling rate to prevent any aliasing from occurring. Measurements are made at 18 discrete downstream locations from $x/D=1$ to $x/D=11$. The setup allows for the transducers to be pulled outward to follow the expansion of the shear layer as the ring is traversed downstream for each individual measurement location. The probes are positioned 10 mm away (in the radial direction) from the outer edge of the shear layer, close enough to sense the hydrodynamic pressure fluctuations from the near field and far enough from the highest non-linear hydrodynamic fluctuations to prevent intrusion in the flow and saturation of the signal. This position also ensures sensing of the strong acoustic fluctuations that emanate from the jet.
flow before they decay as they propagate to the far-field. A very tedious and time-consuming process ensures consistency in the orientation of each probe to make sure that the individual outputs are similar in magnitude and frequency content and that their position around the array is symmetric with respect to the jet centerline at each streamwise location.

- The jet’s far-field acoustics are measured along an array of 6 G.R.A.S. (type 40BE) 1/4 inch pre-polarized, free-field condenser microphones. Each individual microphone is positioned at 75 jet diameters from the center of the jet exit and placed at polar angles from the jet centerline axis (x) ranging from $\phi = 15^\circ$ to $\phi = 90^\circ$ with $\Delta \phi = 15^\circ$ increments (see Figure 2.2 for a schematic of the setup within the chamber). These different positions allow for a measure of the directivity of the jet’s acoustics. The frequency response of the microphones is 10Hz-40kHz (± 1 dB) and the dynamic range is 166 dB ref. 20 µPa.

Figure 3.2: Location of all far-field and near-field pressure measurement points.
All near-field and far-field pressure channels are low-pass filtered at 20.48 kHz
and then sampled at 40.96 kHz using a National Instruments PXI system equipped
with three 8-channel NI-4472 A/D converting boards with 24 bits resolution and
8 dedicated low-pass filters. Figure 3.2 shows a top view of the location of all the
near-field and far-field measurement points. The outer edge of the axisymmetric
shear layer is also shown, to give a sense of the distance separating the jet flow
from the far-field microphones.

3.3 Results

3.3.1 Near- and far-field pressure spectra

Hereinafter, the subscript \( n \) will denote the near-field pressure and the subscript
\( f \) will denote the far-field pressure. Given two signals \( p_n \) and \( p_f \), the single-sided
cross spectral density function defined for \( 0 < f < f_s/2 \) by:

\[
G_{nf}(f) = \frac{2}{T} \langle \hat{p}_n(f) \hat{p}_f^*(f) \rangle
\]

where \( T \) is the block length and \( \langle . \rangle \) denotes the ensemble average over the total
number of blocks. The asterisk denotes the complex conjugate and the Fourier
transform \( \hat{p}(f) \) of \( p(t) \) is given by:

\[
\hat{p}(f) = \int_0^T p(t) e^{-i2\pi ft} dt
\]

In the same manner, the auto spectral density functions are estimated with:

\[
G_{nn}(f) = \frac{2}{T} \langle \hat{p}_n(f) \hat{p}_n^*(f) \rangle
\]

All the auto-spectra are checked by comparing the total mean square value and
the integral of the auto spectral density function, which should be equal:

\[
\overline{p_n^2} = \int_0^{f_s/2} G_{nn}(f) df
\]
where the over-line denotes the time average.

Figure 3.3: (a) Far-field auto-spectra, (b) compensated, (c) smoothed and (d) as a function of Strouhal number.

The different near-field pressure sensors are identified by their azimuthal position $\theta$ as defined in Figure 3.1 and the downstream position of the run $x/D$. The far-field microphones are identified by their polar angle $\phi$ with respect to the jet centerline. The far-field pressure power spectra, $G_{ff}(f)$, are shown in Figure 3.3 along with the compensated spectra $f \times G_{ff}(f)$ to identify the dominant frequencies on the logarithmic scale [64]. Additionally the data is smoothed using a $\pm 10\%$ moving average for clarity of the presentation as shown (bottom left) and the data is plotted as a function of Strouhal number (bottom right) for dimensionless com-
parison with other studies on different jets. All the spectra were computed using 300 ensembles of $n_s=8192$ samples resulting in a frequency resolution of $f_s/n_s=5$ Hz. As the polar angle increases from $\phi = 15^\circ$ to $\phi = 90^\circ$, the spectrum of the far-field sound becomes more broadband but considerably lower in magnitude. This is consistent with the known directional nature of the jet acoustic pressure field [74]. Tam & Chen [98] argued that the weaker and more broadband pressure fluctuations at large polar angles are due to fine-scale turbulence in the jet and the larger, more narrow band fluctuations at small angles from the jet axis corresponds to mixing noise caused by the large-scale vortex structures interacting in the flow.

The sound pressure level is shown in Figure 3.4 and is computed using:

$$SPL_{ff}(f) = 10 \times \log \left( \frac{G_{ff}(f)}{(20 \times 10^{-6})^2} \right)$$

(3.5)

It is observed that the Kulite pressure sensors present a quite inconsistent frequency response above 5000 Hz, therefore no physical interpretation of the near-field pressure data will be given concerning the higher frequency content of the
signal. The frequencies below 5000 Hz are however very well resolved and Figure 3.5 (a) shows very good azimuthal symmetry in the magnitude and frequency content of each sensor. As the flow evolves downstream (Figure 3.5 (c) and (d)), the dominant frequencies band decreases from around 3000 Hz (St=0.5) at x/D=1 down to around 600 Hz (St=0.1) at x/D=11, with a Strouhal of 0.3 towards the end of the potential core (x/D=6), which is consistent with the findings by Crow & Champagne [34] who showed that the preferred orderly mode to reach saturation at the end of the potential core in the jet was at a Strouhal number of 0.3.
3.3.2 Sound source location

In high Reynolds number turbulent axisymmetric jets, three distinct regions have been identified to account for the different dynamics observed in the flow as it develops from the nozzle exit to the downstream region:

1) The mixing layer region, where the axisymmetric shear layer develops instabilities around the potential core region defined as the region where the velocity is equal to 99% of the exit centerline velocity. The growth of the jet shear layer in this region is linear and the mean velocity profiles and RMS fluctuating velocity profiles can be collapsed onto a similarity profile [105]. It is in this region that the turbulence levels are most intense.

2) The collapse of the potential core region is a transition region between two different self-similar regions. The characteristic frequency (or most amplified frequency) in this region has been shown [34] to be in the vicinity of St=0.3. Some variability on this result has been found but differences in experimental setups and techniques of measurement of this frequency were found to be responsible for the variability. Previous experimental and theoretical work has shown that the part of the jet that follows the collapse of the potential core, where the highest turbulence intensities are observed, is the region where the most intense sources of noise exist. It is in this region that the axisymmetric shear layer meets at the centerline and noise is produced by turbulent eddies and large scale structures interacting, merging and then breaking down into smaller scale turbulence. Through experimental measurements, Glauser et al. [46] devised a low-dimensional model often referred to as the “volcano” event (Figure 1.2) where a large-scale low-momentum vortex ring is overcome by a high velocity small scale vortex ring, breaking it up into higher order structures in a short time, highly intense event.

3) The far self-similar region is where the now fully developed flow obeys a similarity profile that remains valid until many diameters downstream. The growth
of the jet shear layer in this region is also linear but the slope is slightly steeper than in the mixing layer region.

Our experiment was therefore designed in an attempt to further understand how the near-field pressure around the jet flow relates to this type of higher order event and how it relates to the far-field sound generated by these events. By sampling the pressure around the jet shear layer at a given downstream position simultaneously with the sound in the far-field, it is then possible to compute the cross-correlation between these signals to detect both the magnitude and phase of the correlation. From the normalized magnitude can be interpreted how well the information carried by the hydrodynamics-dominated pressure near-field is propagated to the far-field through acoustic pressure waves. Using the phase information, one can extract the speed at which the information is propagated, which should be the speed of sound if the information in fact travels through sound waves.

In general, the cross-correlation function between two signals \( f \) and \( g \) is given by:

\[
R_{fg}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T f(t)g(t + \tau)dt
\] (3.6)

In practice, and in the present case, the cross-correlation between a near-field pressure sensor and a far-field microphone is computed in discrete form as follows:

\[
R_{nf}(\tau) = \overline{p_n(t).p_f(t + \tau)}
\] (3.7)

with the over-line denoting time average. In the same way, the cross-correlation between two pressure sensors at different azimuthal locations around the near-field holder is given by:

\[
R_{nn}(\Delta \theta, \tau) = \overline{p_n(\theta, t).p_n(\theta + \Delta \theta, t + \tau)}
\] (3.8)

with \( \Delta \theta \) the azimuthal separation between the sensors. These quantities can be expressed in a normalized form as follows:
\[ \rho_{nf}(\tau) = \frac{p_n(t) \cdot p_f(t + \tau)}{\sigma_n \sigma_f} \]  

\[ \rho_{nn}(\Delta \theta, \tau) = \frac{p_n(\theta, t) \cdot p_n(\theta + \Delta \theta, t + \tau)}{\sigma_n^2} \]  

(3.9)  

(3.10)

with \( \sigma_n \) and \( \sigma_f \) the standard deviation of the near- and far-field signals respectively. Figure 3.6 shows the normalized cross-correlation \( \rho_{nn} \) between the near-field sensors for all downstream positions, with the lowest correlation corresponding naturally to a 180° separation and a level of correlation above 20% at ±60° separation.

![Figure 3.6: Normalized cross-correlation between the near-field pressure sensors as a function of angular separation for all downstream positions.](image)

These quantities are in practice computed using the inverse fourier transform (IFT) of the cross-spectral density function between the two signals. Indeed the result is mathematically identical but the computation is much faster in the frequency domain since the number of operations performed is proportional to \( N \cdot \log(N) \) (with \( N \) the number of data points) instead of \( N^2 \) in the time domain. The cross-
correlation function is therefore in practice calculated using:

\[ R_{nf}(\tau) = \int_{-\infty}^{\infty} S_{nf}(f) e^{2\pi i f \tau} df \]  

(3.11)

with \( S_{nf} \) the two-sided cross spectral density function defined for \( 0 < f < f_s \) by:

\[ S_{nf}(f) = \frac{1}{T} \langle \hat{p}_n(f) \hat{p}_f^*(f) \rangle \]  

(3.12)

Figure 3.7 shows the normalized cross-correlation of the near-field pressure sensor located at \( \theta = 96^\circ \) (see Figure 3.1) with each far-field microphone for all downstream positions of the ring array. As can be seen, the highest correlation magnitudes occur with the microphone located at \( \phi = 15^\circ \) from the jet centerline, showing the directivity of the jet acoustics. This finding is consistent with the literature [74] and theory, the jet acting as a dipole sound source with highest sound intensity at angles closer to the main axis. A second feature clearly noticeable for all far-field microphones is that the magnitude of the correlation increases for measurements from \( x/D = 1 \) to \( x/D = 8 \) and then decreases until the last downstream measurement at \( x/D = 11 \). Figure 3.8 shows the correlation with microphone 6 (at \( \phi = 15^\circ \)) only. A higher correlation between the near-field dynamics and the far-field sound means that the events that occur in the flow at a given time leave a signature in the near pressure field that efficiently translates into a sound pressure wave that is then sensed at a time \( \tau \) later. This interpretation is based on an average over all frequencies. To study the contribution of each frequency to the far-field, one has to compute the cross-spectrum or the coherence spectrum between the two signals, which is a study left for a future investigation. We are here looking at the downstream location where the highest correlations with the far-field lie (over all frequencies). The phase shift seen in this figure for a given microphone as the near-field measurement is moved downstream is only due to the distance between the two measurement locations and a traveling velocity can be retrieved from this time delay knowing the distance between the measurement
Figure 3.7: Normalized cross-correlation between the near-field pressure sensor at \( \theta = 96^\circ \) for all downstream positions and each of the far-field microphones from \( \phi = 90^\circ \) (mic 1) to \( \phi = 15^\circ \) (mic 6).

locations. A more accurate way of studying the phase information is to extract the angle function from the cross-spectral density. Since the cross-correlation function \( R_{nf} \) is not an even function, the Fourier transform of this quantity \( S_{nf} \) is a complex function that can be written:
Figure 3.8: Normalized cross-correlation between the near-field pressure sensor at \( \theta = 96^\circ \) for all downstream positions and the far-field microphone at \( \phi = 15^\circ \) (mic 6).

\[
S_{nf}(f) = C_{nf} + iQ_{nf}
\]  

(3.13)

where \( i^2 = -1 \), \( C_{nf} \) the coincident spectral density function and \( Q_{nf} \) the quadrature spectral density function. From these quantities, the spectrum can be separated into a magnitude and an angle as follows:

\[
|S_{nf}(f)| = \sqrt{C_{nf}^2 + Q_{nf}^2}
\]  

(3.14)

the angle being defined as:

\[
\theta_{nf} = \arctan \left( \frac{Q_{nf}}{C_{nf}} \right)
\]  

(3.15)

In the frequency range where the signals are coherent, the phase angle plot should show a linear evolution which represents the time delay \( \tau \) for the signal to travel from the near-field to the far-field. This time delay is given by:
\[ \tau = \frac{m}{2\pi} \]  

(3.16)

where \( m \) (in rad/Hz or rad.s) is the slope of the linear part of the curve. Using the known distance between the measurement locations one can extract the traveling velocity of the correlated information. This characteristic velocity is expected to be equal to the speed of sound in air since the information should be carried to the far-field through sound pressure waves, this is at least the assumption and it can be verified by analyzing this quantity. This also constitutes a relevant verification to make sure that the far-field instrumentation is in fact really in the far-field, i.e. that no information is carried to the microphones by hydrodynamic pressure fluctuations or information convected by the flow. This type of concern would arise more likely with the sensor closest to the flow (i.e. microphone 6 at \( \phi = 15^\circ \)).

The phase-angle of the cross-spectrum of the near-field pressure sensor at \( \theta = 72^\circ \) and the far-field pressure sensor at \( \phi = 60^\circ \) and \( x/D = 5 \) is plotted in Figure 3.9 (top). From the slope of the linear part of the curve and the distance between measurement locations, a velocity of 344.06 m/s is computed. This is satisfyingly close to the speed of sound at the temperature the experiment was carried out \( (c_o = 343.11 \text{ m/s}) \). There are however uncertainties in this estimate, the major one arising from the uncertainty in slope estimation from the phase-angle plots, which is estimated to cause an uncertainty of \( \pm 5 \text{ m/s} \) in the calculation of the traveling velocity. Figure 3.9 (bottom) shows the phase-angle of the cross-spectrum of the near-field pressure sensor at \( \theta = 72^\circ \) and the far-field pressure sensor at \( \phi = 15^\circ \) and \( x/D = 5 \). Proceeding in the same manner, a traveling velocity of 359.91 \( \pm 5 \text{ m/s} \) is calculated, which shows that the information traveling to this sensor is acoustic in nature and that the flow does not convect any information to that far-field sensor, or it is at least not sensed by the sensor. The following table shows estimates of traveling velocities from the near-field pressure sensor at
Figure 3.9: Phase-angle of the cross-spectrum between near-field pressure sensor at $\theta = 72^\circ$ and far-field microphone at (top) $\phi = 60^\circ$ and (bottom) $\phi = 15^\circ$.

$\theta = 72^\circ$ to 3 different far-field microphones at $\phi = 90^\circ$, $\phi = 60^\circ$ and $\phi = 15^\circ$. The velocities range between about 320 and 370 m/s which seems to coincide with the range of the speed of sound, the variations are believed to be due to interactions and refraction within the flow. It is however clear that even with microphone 6 at
\( \phi = 15^\circ \), the information is not carried by convection through the flow, the highest velocities in the flow being around 291 m/s at the nozzle exit. The plots from which were calculated the traveling velocities in the following table are shown in Appendix A.

\[
\begin{array}{|c|c|c|c|}
\hline
x/D & mic. 1 & mic. 3 & mic. 6 \\
\hline
1 & 339.80 & — & 325.91 \\
2 & 348.19 & 342.40 & 346.66 \\
3 & 340.85 & 342.80 & 348.99 \\
4 & 338.13 & 340.44 & 350.14 \\
5 & 335.13 & 344.06 & 359.91 \\
6 & 329.63 & 345.25 & 361.07 \\
7 & 328.91 & 334.68 & 362.44 \\
8 & 334.44 & 338.78 & 359.68 \\
9 & 324.04 & 341.11 & 372.03 \\
10 & 320.70 & 363.35 & 362.54 \\
11 & 313.55 & 372.66 & 342.93 \\
\hline
\end{array}
\]

Table 3.1: Traveling speed (in m/s) of the information from the near-field sensor at \( \theta = 72^\circ \) to the far-field microphones 1, 3 and 6 at \( \phi = 90^\circ \), \( \phi = 60^\circ \) and \( \phi = 15^\circ \) respectively.

From Figure 3.7 we can extract the maximum value of the correlation with each microphone and plot it versus downstream position, which is shown in Figure 3.10. It is clear that for all far-field microphones, the highest intensity of sound comes from the region between \( x/D=6 \) and \( x/D=10 \). Indeed, the normalized cross-correlation coefficient, as defined by Equation 3.9, reaches a quite significant maximum of almost 35\% with the microphone closest to the jet axis (mic. 6). At the steep angles with respect to the jet centerline (\( \phi = 60^\circ \), \( \phi = 75^\circ \) and \( \phi = 90^\circ \)), the
Figure 3.10: Maximum normalized cross-correlation between the near-field and far-field pressure as a function of downstream position, for all microphones mic 1 ($\phi = 90^\circ$) to mic 6 ($\phi = 15^\circ$).

correlation drops significantly and remains below a level of 10%, which is not unexpected since the jet exhibits strong acoustic directivity towards the jet centerline.

Let us now fix the downstream location at $x/D=7$ and look at the correlation of all the pressure sensors around the near-field ring with the far-field microphones. Figure 3.11 shows for each far-field microphone separately the normalized cross-correlation coefficient with all 15 azimuthal pressure sensors. Figure 3.12 shows the correlations with microphone 6 only. The striking feature is that there is no phase delay between the different sensors, which means that events that correlate well with the far-field sound correlate simultaneously with all pressure sensors around the jet. One could have expected that two opposite pressure sensors on the array would have a delay between their correlation peaks with the far-field to account
Figure 3.11: Normalized cross-correlation between all near-field pressure sensors and each of the far-field microphones from $\phi = 90^\circ$ (mic 1) to $\phi = 15^\circ$ (mic 6), at $x/D=7$.

for the different distance to the far-field sensor. This suggests that the near-field pressure fluctuations that translate efficiently into radiating sound are effectively axisymmetric in nature. An azimuthal decomposition into Fourier modes follows to study the contribution of each azimuthal mode to the correlations.
Figure 3.12: Normalized cross-correlation between all near-field pressure sensors and the far-field microphone at $\phi = 15^\circ$ (mic 6), at $x/D=7$.

3.3.3 Modal description of the pressure

The fifteen near-field pressure sensors are equally spaced with an azimuthal separation of $24^\circ$ and the holder and sensors themselves were carefully placed at each downstream position to be located at $1 \text{ cm}$ from the outer-edge of the jet shear-layer. The pressure in the near-field of the jet flow is known to be low-dimensional as shown, among others by Tinney [102] and also Iqbal and Thomas [57] and this feature is again verified in this experiment; unlike the high-speed flow itself that exhibits much energy in the higher azimuthal modes (5, 6, 7) near the collapse of the potential core. It therefore is evident that only the largest, azimuthally coherent structures are able to propagate a hydrodynamic pressure wave through the jet shear layer and results show that the correlation between this pressure wave and the far-field sound is significant, as will be shown in this section. Since the near-field pressure measurement grid is circular and centered on the jet flow centerline, it is natural to decompose the instantaneous signal into azimuthal Fourier modes.
by applying a spatial Fourier transform on the instantaneous measurements. How-
ever, the conclusions on the physical interpretation of the azimuthal modes need to
be carefully drawn since it is just a mathematical decomposition that solely gives a
sense of the dimensionality of the flow structures but does not describe individual
structures, i.e. each mode does not represent a physical structure on its own. The
azimuthal spectrum of the near-field pressure $B_{nn}$ is calculated as follows:

$$B_{nn} = \frac{1}{2\pi} \int_{0}^{2\pi} R_{nn}(\Delta \theta, \tau = 0)e^{-2i\pi m \Delta \theta} d\Delta \theta$$

(3.17)

where $\Delta \theta$ is the azimuthal separation between the sensors and $R_{nn}$ is the cross-
correlation function as defined in Equation 3.8. The single-sided azimuthal spectra
of the near-field pressure at all downstream positions are shown in Figures 3.13
and 3.14. In the present case, since there are 15 sensors equally spaced around
the circular holder, the total number of azimuthal modes is 8 (modes 0 through 7). It is important to note that, to show a fair comparison of the energy in each
mode, for $m > 0$, the modes $+m$ and $-m$ (which are complex conjugates) need to
be added together. This is performed when showing the single-sided spectra. It is
clear that the distribution of energy is low-dimensional. On average, 90% of the
energy is contained within the first 3 azimuthal modes (0, 1 and 2).

Figure 3.15 shows the evolution of the energy content in azimuthal modes 0 to
4 as a function of downstream position, along with the total energy in all modes.
This shows that, in an averaged sense, both modes 0 and 1 seem to be as important
in their energy content and it would be incorrect to conclude that one or the other
is a “dominant” structure of the near-field pressure.

To further understand the contribution of the first modes to the pressure in an
instantaneous sense, the 15 pressure signals are azimuthally Fourier transformed
at every time, the unwanted azimuthal modes were zero-padded and finally the
Figure 3.13: Energy contained in all azimuthal Fourier modes (0 to 7) for downstream positions $x/D=1$ to $x/D=5$.

The signal was inverse Fourier transformed back into the spatial domain to show a time series of the “Fourier mode-filtered” instantaneous pressure. The thus derived low-dimensional representation of the pressure time trace is given by:

$$\tilde{p}(\theta, t) = \int_{-\infty}^{\infty} \hat{p}(m, t)e^{im\theta}dm$$

(3.18)

with $\hat{p}(m, t)$ the pressure signal that has been mode-filtered to only exhibit features of the modes of interest. Figure 3.16 thus shows a sample time series of the
Figure 3.14: Energy contained in all azimuthal Fourier modes (0 to 7) for downstream positions $x/D=1.5$ to $x/D=11$.

pressure at $\theta = 96^\circ$ (in blue) compared to the mode 0 part of the pressure only at the top, the mode 1 part of the pressure only in the middle and the sum of both contributions at the bottom. This shows that even in an instantaneous sense, both modes are as important to satisfyingly reconstruct the pressure signal. This once again shows that an interpretation of the azimuthal modes as structures is not feasible since no independence of behavior between them is observed.

The near-field pressure being now decomposed for all times into azimuthal
Figure 3.15: Comparison of the level of energy in azimuthal Fourier modes 0 to 4 as a function of downstream position.

Figure 3.15: Comparison of the level of energy in azimuthal Fourier modes 0 to 4 as a function of downstream position.

Modes and a “mode-filtered” reconstruction of the pressure signals having been made, it is now possible to correlate the filtered pressure signals with the far-field (instead of the full pressure as in Figure 3.7) to examine which constitutive features of the near-field pressure best correlate with the far-field sound, in an attempt to better understand the propagation of the near-field pressure as sound waves. The time series of the filtered near-field pressure at θ = 96° is correlated with the simultaneously sampled far-field sound at φ = 30°. Figure 3.17 shows the quite interesting result that the pressure filtered to retain only the axisymmetric (mode 0) contribution correlates better (34%) than the full pressure signal (23%). Concurrently, the mode 1 contribution to the near-field pressure correlates very poorly with the far-field, indicating that the antisymmetric (mode 1) part of the pressure does not propagate efficiently to the far-field. The axisymmetric mode 0 is the most efficient propagator to the far-field, it can thus be referred to as the “loud” mode, as can mode 1 be referred to as the “quiet” mode. This is also true for all downstream positions, as shown in Figure 3.18, where the maximum of the
cross-correlation is plotted as a function of downstream distance. Especially in the loud region (between x/D=6 and x/D=10) does the loud mode correlate the best with the far-field sound. This finding is quite insightful and can guide flow control strategies. Indeed, it seems like an active manipulation of the near-pressure field could be sufficient to inhibit the propagation of sound to the far-field. The herein proposed closed-loop control strategy is to actively force the near-pressure field (not the whole jet flow) with an antisymmetric distribution (mode 1) to counteract the axisymmetric nature of the field whenever it is dominant.
Figure 3.17: Comparison of the normalized cross-correlation between the far field sound at $\phi = 30^\circ$ and the mode-filtered near-field pressure at $\theta = 96^\circ$, $x/D=8$.

3.4 Conclusions

From this first experiment relating the near-field pressure to the far-field sound, several original findings have been made:

- The location of the most efficient sound producing region of the jet is found to be between $x/D=6$ and $x/D=10$ at the Mach number of 0.85, after the collapse of the potential core.

- The near-field pressure is truly low-dimensional (unlike the flow it surrounds) with a dominance of azimuthal modes 0 and 1. It was shown that both in an average sense (azimuthal spectra) and in an instantaneous sense (mode-filtered time series), modes 0 and 1 are needed to accurately rebuild the pressure. No separation of the two modes can be performed to interpret each of them as an individual structure of the near-field pressure, they are
It is shown however that only the axisymmetric component of the near-field pressure correlates well with the far-field sound and furthermore that the mode 1 contribution inhibits sound propagation. From this finding an active flow control strategy is conceptually described where sensing of the instantaneous azimuthal content of the near-field pressure can be an input for closed-loop actuation at the lip of the jet to anti-symmetrically force the near pressure field only (not the flow since it would require too much energy) using the time-resolved input from the sensors. An active flow control actuating system has therefore been designed to be able to slightly manipulate the outcoming shear-layer and force the outer edge using different azimuthal modes (see Chapter 5).
Chapter 4

Dynamical system of the loud region of the jet

Several attempts have been made at investigating the dynamics of the high-speed jet, that is to say to quantitatively and in a time-resolved manner measure flow quantities so as to capture from the smallest to the largest time-scale events in the flow. From this type of experimental or computational measurement can be extracted dynamical information from which much can be learnt as to the types of coherent processes that occur and how they relate to the far-field sound. Since the 1970's [20] [34], it is widely recognized that even very high Reynolds number flows contain coherent structures (see also Hussain [56]) that carry distinctive features such as organized vorticity, and that exhibit repeating patterns of turbulent motion. Unlike the previous belief that turbulent flows were dominated by random motion and chaos, through advanced measurements, simulations and analytical data processing tools, it is now quite easy to recognize such features in high-speed flows. The problem still lies in how to physically interpret these results to explain the mechanisms and build models of these flows. Such models are important to the field since numerically solving the Navier-Stokes equations in high Reynolds number flows is not even conceivable in the foreseeable future of computing evolution. Models can therefore improve on Reynolds Averaged Navier-Stokes (RANS) models or validate Large Eddy Simulations (LES) of such flows. Direct Numerical Simulations (DNS) of high-speed jets performed at very low Reynolds
number [40] along with computations of the instantaneous aeroacoustic field have
provided great insight into the dynamics of these flows and their relation to the
far-field sound. Freund et al. [109] have applied flow control using adjoint meth-
ods in their DNS simulation of a two dimensional high-speed jet with the control
objective of minimizing the far-field sound. The control is performed in a “black-
box” near the jet exit and modifications to the control input are made iteratively
at retarded times to slightly modify the dynamics of the flow and minimize the
sound propagated to the far-field. A quiet jet was in this manner simulated and
compared to the uncontrolled loud jet. Quite surprisingly, a simple comparison
of the instantaneous dynamics of the controlled to the uncontrolled flow does not
show a significant difference in the organization of the large-scale structures of the
flow, which indicates that the noise production mechanism is quite subtle. Only
once a processing of the data using proper orthogonal decomposition (POD) is
performed does a difference in spatial phase appear between the first modes. This
type of study of the dynamics of jets shows that a similarly subtle control strategy
needs to found when such a slight change in the flow has such a great impact on
the far-field sound.

Early experimental measurements of the dynamics of high-speed jets were per-
formed using rakes of hot-wires along with hot-wire anemometers. From such
measurements are calculated multi-point correlations of velocity components and
with assumptions of homogeneity and periodicity in the jet flow, low-dimensional
techniques like the POD could be applied [44]. Citriniti [28] took measurements of
the dynamics of the axisymmetric incompressible jet at one downstream position
using an in-house designed rake of 138 hot-wires. The experiment was then carried
out at multiple near-field positions by Gamard [42] and multiple far-field positions
by Jung [59] to study the evolution of the azimuthal modes as the jet develops
downstream. Computational efforts towards measuring the dynamics of axisym-
metric jet flows are today feasible at rather low Reynolds numbers ($\simeq 5 \times 10^3$)
in direct simulations [40] and at intermediate Reynolds numbers ($\simeq 5 \times 10^4$) us-
ing LES [15]. These investigations have the advantage of containing time-resolved information on all flow quantities simultaneously (velocity, density, acoustics, hydrodynamics, vorticity, etc) from which much valuable information can be gained. However the limitations are the rather low Reynolds numbers attainable and the small amount of time series computed. Experiments and simulations will therefore need to feed each other for many years to come (thankfully!).

4.1 Measuring flow dynamics

Experimental time-resolved measurements of turbulent flow fields certainly emerged with the hot-wire anemometry technique, the principles of which date back to the beginning of the twentieth century. Comte-Bellot [31] provides a thorough historical and technical perspective on the way hot-wires enabled single-point measurements of statistical turbulent quantities. Much later with the advent of lasers was developed the laser Doppler anemometry (LDA), which although technically much more challenging and costly for the experimenter, presents the advantage of being a non-intrusive technique, meaning that the flow will remain undisturbed by the measurement. It is now possible to obtain 3-component single-point measurements of velocity in high-speed flows. The main issue with this type of measurement being seeding, since it is based on the Doppler shift provoked by a particle crossing the measurement volume and scattering light with a frequency shift proportional to its velocity. A review on the technique and many details on the theory behind LDA can be found in Buchave et al. [21], the authors of this article having contributed most of the pioneering work related to this experimental technique. Non-intrusive multi-point velocity measurements were only developed toward the late 1980s with the work of Adrian [1] when high power lasers and fast response cameras were combined to form particle image velocimetry (PIV). This technique now enables accurate, well-resolved velocity measurements simultaneously in a whole plane but
at discrete times, that is to say not in a time-resolved manner. Much can be learnt from such measurements if one has the desire to study the statistical properties of a flow such as mean velocity profiles, phase-averaged velocity fields or simply collect databases of instantaneous snapshots of velocity fields to then perform multi-point spatial correlations and other post-processing techniques. Since this type of measurement lacks the ability to perform time-resolved measurements, a new interest in hot-wire anemometry emerged with the design of rakes of hot-wires to measure the dynamics of flow fields in whole planes. Citriniti and George designed a rake of 138 hot-wires to investigate the azimuthal modal distribution of the axisymmetric jet and were able to sample the jet dynamics in a time-resolved manner with some carefully accounted for spatial filtering due to the coarseness of the hot-wire rake. Unfortunately, only a few research groups have had the patience and dedication to design and calibrate such a piece of equipment. This kind of technique is very effective but limiting in terms of flow velocity since it is an intrusive technique and the disturbance is accentuated at high speeds, not to mention the probe needs to be able to minimize turbulent vibrations and withstand the high dynamic pressure of high-speed flows. The next step in dynamical measurements of such flows was the recent development of the multi-plane PIV measurement technique [61] which presents the advantage of being a non-intrusive technique applicable to any type of flow, whether it is low or high speed, in air or water and it enables a direct measurement of the three-component acceleration field. Depending on the type of flow and the way it is performed, Eulerian or Lagrangian acceleration can be measured by finite difference calculation of the consecutive PIV measurements, the second one being delayed by a carefully chosen time lapse that is tied to the time scales of the flow under consideration. This issue will be treated in more detail in the following sections.

Experimentally measuring the dynamics of the high-speed jet flow is a quite challenging task. We are today experimentally limited to two-dimensional plane
measurements (3 components) since holographic PIV techniques for volume measurements are still in the early stages of development [114] and are limited to small volumes, low spatial resolution and rather low-speed flows. However, planar measurements can be very informative, especially when all three components of velocity can be measured simultaneously on a relatively fine grid such as when using stereoscopic particle image velocimetry (PIV). This optics-based measurement technique is interesting not only because of its non-intrusiveness (the flow remaining undisturbed by the measurement) but also because it is not conceivable to use rakes of hot-wires in high-speed compressible flows. PIV is therefore the only experimental instrument capable of providing planar velocity data with a fine resolution in high-speed flows such as the axisymmetric jet. PIV measurements in a longitudinal plane (x,y) can be performed to study the evolution of the structures as they convect in the shear layer. However, streamwise vorticity is an important quantity in the jet flow, being responsible for much of the dynamics and found to be linked to noise production [48]. It was therefore chosen in the present study to perform the measurements in the cross-flow plane of the jet, i.e. the plane perpendicular to the jet centerline, and to move the measurement plane downstream by discrete increments to cover a measurement volume extending from the near jet exit (x/D=3) to beyond the region of collapse of the potential core which occurs between 5 and 6 jet diameters downstream. At this location, turbulence intensity reaches a maximum and the near-field pressure dynamics seem to relate best with the far-field sound, as shown by Hall et al. [52]. To capture and rebuild the dynamics of the flow, a low-dimensional experimental approach is taken here, in which a large database of the acceleration field (and thereby the velocity field) is needed. Since the velocity data contains the information described by the Navier-Stokes equations, a system of ordinary differential equations (ODE’s) is fit to the data to provide a dynamical system capable of predicting the low order dynamics of the jet flow field, given an initial condition. This capability carries interest for flow control applications where a dynamical estimate of the state of the flow can be of
great use in conjunction with a direct measurement.

4.2 Dual-time particle image velocimetry instrumentation

4.2.1 Jet facility and flow conditions

This second experiment is carried out in the same facility as the first experiment, the Syracuse University Skytop Anechoic Chamber. In this experiment, the jet flow is set at a Mach number of 0.6 to maximize the running time. The air is supplied by the 100 hp 2-stage compressor which fills 5 tanks to a pressure of 3.1 MPa (31 Bars or 450 Psi), with a total capacity of 31 m$^3$. This amount of compressed air enables runs of 35 minutes long at a Mach number of 0.6. This running time is decreased to 15 minutes at a Mach number of 0.85 which does not provide enough time to run a whole set of PIV data. Additionally, the higher speed presents more challenges for the dual-time PIV timing and measurement accuracy. Indeed, a higher flow velocity requires the delay between the systems to be much smaller and the relative error between the two systems becomes comparable to the change in velocity, which leaves the measurement with a measure of the error between the systems instead of the quantity we are interested in capturing.

The flow remains significantly compressible at Mach 0.6 and the conclusions from this study can be safely considered valid for high subsonic Mach number cases. Moreover, the sound level remains high enough to compute correlations between near- and far-field quantities. The experiment was run during the months of February and March, when temperatures in the Central New York area rarely break the thirty degree Fahrenheit level. For this reason and for a matter of consistency and repeatability throughout the experiment, the make-up-air unit provides room temperature heated air (20°C) through a plenum chamber and into the anechoic chamber around the core high-speed flow with a 3 m/s co-flow. The co-flow also fa-
cilitates measurements in the entrainment region of the jet since seeding is needed for particle image velocimetry measurements even in regions of velocities close to zero. An acoustically treated centrifugal eductor fan pulls the air brought by the jet and the co-flow out of the chamber to balance the pressure in the latter. The anechoic chamber is padded with high density fiberglass wedges with a high-pass cutoff frequency of 150 Hz. An acoustic calibration of the chamber was performed by Peguero [84]. The main acoustic features of the high speed jet are located in a broad bandwidth around 1000 Hz [102]. Extensive details on the design of this newly refurbished jet facility can be found in Tinney et al. [104]. The first experimental measurements on this jet were performed by Charles Tinney [102] and André Hall [51]. The present measurements are found to be statistically very consistent with the initial ones performed on the jet which adds to the confidence in the quality of the database. The Reynolds number of this turbulent compressible flow is 690,000 based on a nozzle diameter of 50.8 mm.

4.2.2 Background on the technique

Particle image velocimetry (PIV) is a now widespread experimental fluid measurement tool that enables accurate and spatially well resolved measurements of the 3 components of velocity in a whole plane simultaneously. The pioneering use of multiple PIV systems was proposed and later implemented by Kähler and Kompenhans [61] to develop a tool capable of measuring time derivatives of the velocity (i.e. acceleration) as well as spatial derivatives (i.e. velocity gradient tensors, strain rate tensors and vorticity field) in a whole plane. Several research groups have since performed dual-plane PIV [60, 55, 49, 110, 18, 77], using two PIV systems for measurements in two parallel and adjacent planes; as well as dual-time PIV [110, 86], using both systems in the same measurement plane separated in time by a short delay. The latter technique enables measurement of acceleration or more generally of the time derivative of the velocity field which, associated
with the proper orthogonal decomposition (POD), enables dynamical systems de-
velopment with a potential use for closed-loop flow control. The physical quantity
measured using DT-PIV is in all respects the equivalent of that measured using a
time-resolved system (TR-PIV or hot-wire anemometer) except that measurements
are taken only at discrete independent times, where a time-resolved measurement
would have an estimate of that quantity at all times in a time-resolved manner,
meaning that the entire spectrum of the dynamics is measured. In the present
configuration where the consecutive PIV measurements are performed at the same
location, the Eulerian acceleration (or local acceleration) is computed directly. The
Lagrangian acceleration (or material acceleration) is the whole left-hand side of
the Navier-Stokes equations defined as:

\[
\frac{D\ddot{u}}{Dt} = \frac{\partial \ddot{u}}{\partial t} + (\ddot{u} \cdot \nabla)\ddot{u}
\]

(4.1)

the first term on the right-hand side being the Eulerian acceleration and the sec-
ond term being the non-linear convective acceleration. To measure this quantity
experimentally, one has to follow each particle as it moves through the velocity
field to take into account both contributions to the acceleration of the particle: the
unsteadiness of the velocity field and the change in velocity as the particle moves
through this field. Lui and Katz [68] have performed such measurements using a
four exposure PIV system (i.e. two PIV systems) and an algorithm to follow the
particle and compute the whole in-plane Lagrangian acceleration term. This tech-
nique was applied in order to estimate from this measurement the instantaneous
pressure field given by:

\[
\nabla p = -\rho \left( \frac{D\ddot{u}}{Dt} - \nu \nabla^2 \ddot{u} \right)
\]

(4.2)

assuming that viscous effects are negligible far from boundaries and assuming
incompressible flow. The pressure field then results from an integration of this
equation over the whole computational domain under these assumptions. Access
to the instantaneous pressure field is of great interest for understanding the physics underlying in a flow since pressure is a quantity integrated over space and therefore contains information from the whole flow and is not just a local quantity like velocity. Unfortunately, this technique is limited to incompressible flows in which \( \rho \) is constant throughout the flow. An instantaneous measurement of the local density would be needed to estimate pressure using this technique in a compressible high-speed flow, which would give access to the acoustic field and could be very beneficial for understanding and localizing sound sources in the jet. The instantaneous pressure field inside the jet would however be very interesting, even at a lower Mach number closer to satisfying the incompressible assumption since the relationship between the high dimensional pressure in the flow and the low dimensional pressure field around the jet flow as commonly measured is still not quite understood. There is a mechanism that filters the pressure waves inside the jet and only lets through a low dimensional pressure field (dominated by azimuthal modes 0 and 1). In this aim, for future studies and to take advantage of the present experimental setup that permits such a measurement, an additional run was performed at a lower Mach number of 0.3 to satisfy incompressibility while maintaining a high Reynolds number.

Christensen and Adrian [27] have developed a similar experimental apparatus called particle image accelerometer (PIA) for measuring the two in-plane components of the Eulerian acceleration using 2 cameras and 4 lasers. From the four PIV pictures, two velocity fields are computed and a finite-differencing technique is used to compute the Eulerian acceleration. The same process is accomplished in the present experiment except that all 3 components of the acceleration field are computed in an entire plane. A study of the accuracy of the method and a quantification of the measurement noise is included in the study by Christensen and Adrian. Three sources of noise contributing to the overall error are mentioned: 1) random error independent of the velocity field and velocity gradients due to
the electronics of the cameras, 2) *bias error*, function of the flow field and due to pixel-locking (or peak-locking) and 3) *gradient noise*, function of the velocity gradient field and that can be caused by a non-uniformity of particle distribution within the interrogation area. A correction to the statistics of the velocity time-derivatives is proposed analyzing how each of these sources of error influences the actual measurement. For future in-depth analysis of the acceleration field of the jet, such an analysis might be beneficial to correct for the measurement errors between the systems. However, in the present case, where the POD is applied to each velocity field from both systems and only the lower dimensional modes are included in the dynamical system, these particular errors are considered to have a minimal impact on the results of the low-order dynamical system (LODS).

In the same way as Perret *et al.* [86] recently performed on a low Reynolds number mixing layer to develop dynamical systems and in a similar fashion as Christensen and Adrian performed except measuring all 3 component of acceleration, two stereoscopic PIV systems are used in the present experiment in an identical cross-flow plane perpendicular to the jet axis. Using finite differencing, local acceleration is computed for each pair of velocity field obtained. The process is reiterated for different downstream positions ranging from 3 to 10 jet diameters so as to obtain a 3-dimensional acceleration field of the high-speed jet. A picture of the setup is shown in Figure 4.1.

### 4.2.3 Seeding

Olive oil droplets are generated by a 12 Laskin-atomizer type nozzle *Pivtec* seeder for use as PIV seeding of the main high-speed jet flow. The characteristic diameter of an olive oil droplet is in the range of 1 to 5 \( \mu m \) which has shown to be small enough to follow the streamlines in all types of flows. The seed is introduced in
the jet nozzle well upstream of the flow straightener so as to minimize the impact on the flow and maximize the mixing of the seed to the jet flow. A second seeder (TSI model 9307) is used to generate oil droplets inside the settling chamber and is entrained inside the anechoic chamber around the main jet flow and pulled out of the chamber through the exhaust system. Co-flow seed is necessary since ambient air is entrained in the shear layer region, where the high-speed flow interacts with the settled ambient air. To be able to capture these dynamics during the experiment, the co-flow seed is introduced several minutes before running the PIV so that the anechoic chamber is uniformly seeded with oil droplets. The high-speed jet is then turned ON followed by the jet seeder, to minimize the accumulation of oil inside the jet nozzle. The concentration of seed is critical to ensure the quality of the PIV database. Too little or too much seeding decreases the capability of the software to calculate reliable cross-correlations between consecutive PIV pictures for a given interrogation area size. An iterative process was implemented to optimize the amount of seeding introduced and guarantee repeatability between runs.

4.2.4 PIV systems

Two identical Dantec Dynamics stereoscopic PIV systems are used in this experiment. This comprises four CCD cameras with 1280 x 1024 pixel sensors and 12 bit resolution giving a large dynamic range of grey scales, which is especially important in a high-speed flow with limited seeding. Two double cavity, 532 nm wavelength, 200 mJ New Wave Research Gemini PIV lasers are used in conjunction with light sheet optics and mirrors to guide the laser path and illuminate the area of interest. All four cameras and two lasers are mounted on a single experimental bench that traverses downstream so as to minimize the number of PIV calibrations performed. Figure 4.1 (b) shows a schematic of the setup. The camera arrangement purposely makes the angle between the cameras in each pair equal to make the measurements
Figure 4.1: (a) Front view and (b) top view of the DT-PIV experimental setup.
from both systems as identical as possible. This minimizes the differences in the level of error which would be amplified when computing the time derivatives. The traverse is a single axis Dantec Dynamics motor/controller system with a 6 \( \mu \text{m} \) resolution. Special care and time was taken in making the four laser sheets identical and aligning them to minimize the error between the measurements of both systems. Each laser contains two separate cavities and Q-switches that let the infrared light out before it reaches manually adjustable mirrors and lenses. The infrared light (1064 nm) from both beams is directed through a second harmonic generator (SHG) to half the wavelength (532 nm) which makes the light green (i.e. visible). The reasons for making the laser light green is because the cameras are more sensitive to that wavelength and also for safety purposes, it is always more prudent to work with lasers in the visible range. Both beams then follow an identical optical path to the sheet generator, therefore any misalignment between the two cavities needs to be adjusted using the mirrors placed before the SHG. More details on the particular procedure of alignment can be found in [79]. The sheet thickness is 3 mm which is thick enough to enable the measurement of the out-of-plane component of velocity (\( u_x \)), which requires that a particle traveling through the light sheet is illuminated by both pulses of the lasers. In the present case, the delay between pulses is set to 5 \( \mu \text{s} \), which allows a particle with a velocity of 200 m/s to move 1 mm within the 3 mm thick laser plane.

### 4.2.5 Polarization of the laser light sheets

In taking double frame images, PIV cameras have the mechanical limitation of having a long second exposure time (\( \approx 111 \text{ ms} \)) relative to the first one (\( \approx 132 \ \mu \text{s} \)). Therefore when using two PIV systems, the light scattered by the particles illuminated by the laser pulses of the second system must be made invisible to the cameras of the first system. Furthermore, when the delay \( \Delta t \) between the
two systems is very small, as it is the case here, the reciprocal must also be true, i.e. the light scattered by the particles illuminated by the laser pulses of the first system must be made invisible to the cameras of the second system. Figure 4.2 explains the timing issue when using two PIV systems. Several techniques can be implemented to perform the task of separating the light of the two lasers. Mullin and Dahm [77] in their measurement of velocity gradients used two light sheets of different wavelengths (i.e. different colors: 635 nm red and 532 nm green) and placed color filters on the cameras. The technique chosen in the present experiment is the more commonly used orthogonally polarized light sheets and linear polarizing lenses mounted on the cameras. The lasers available here have an infrared fundamental beam (1064 nm), the frequency of which is doubled through a second harmonic generator (SHG). A half-wave plate is mounted on the input to the SHG and the SHG crystal is oriented so that the output laser beam (532 nm) is polarized vertically [79]. One laser is therefore mounted horizontally and the second one mounted vertically with respect to the setup base plate as can be seen in Figure 4.1 so that the out-coming laser sheets are polarized orthogonally with respect to one another. The laser sheet from system 1 is hence polarized
vertically while that of system 2 is polarized horizontally. Additionally, each camera is equipped with linear polarizers to let only the desired light through. This technique works in the limit that the particles that scatter the polarized light are spherical and small in size, which is the assumption made here since the olive oil droplets are of order 1-5 $\mu$m diameter, in which case the Mie scattered light emitted from spherical particles remains polarized. In the case where particles are aspherical and of large size, the incoming light will be depolarized as it is scattered off of the particles. This assumption was validated in the present study by setting the delay between the PIV systems to zero ($\Delta t=0$) and the laser power of system 1 to zero. No significant amount of light was captured by the cameras from system 1 nor was any bias in the measurements found due to interference between the two systems.

4.2.6 DT-PIV calibration and image processing.

The velocity fields are computed using the Dantec Dynamics Flow Manager software (version 4.71). A cross-correlation algorithm with an interrogation area size of 32 x 32 pixels has shown to provide the best results. An overlap of 75% between areas is used to increase the effective resolution without compromising accuracy of the measurement. A 5 x 5 moving average is applied to all velocity fields to replace spurious vectors. The origin of these few spurious vectors has several reasons:

1) The area far from the jet core and that doesn’t appear in the final results lacks laser lighting so the data in that border region is hardly reliable.

2) Sporadic ejections of larger oil droplets from the jet lip has shown to create spurious vectors in a small amount of velocity fields.

3) Randomly distributed background noise in the images.

The moving average process replaces each spurious vector by an average of the adjoining vectors. And finally, computation of the 3-component velocity fields is
performed using a direct linear transform (DLT) of the velocity field from both cameras. The DLT is created during the calibration process before the measurements are made. A calibration target that exhibits an origin dot and an array of dots along the y and z direction is positioned very precisely in the plane of measurement given by the location of the laser sheets (x=0). Focusing of the cameras is performed followed by fine-tuning adjustments to ensure that the Scheimpflug condition is verified (i.e. the laser plane, the lens plane and the CCD sensor plane must all meet along one line). Camera focusing has shown to be critical in minimizing the noise in the velocity measurements. Pictures of the target are taken by the two cameras from both systems and the process is repeated with the target at 6 positions around the origin (± 1 mm, ± 2 mm, ± 3 mm). From these images a 3-dimensional DLT is calculated to account for displacements in all three directions. Using this transformation, both velocity fields from cameras A and B (as shown in Figure 4.1) are mapped onto a common interpolated grid to estimate the 3-component velocity field. A resolution of 1.8 mm in both directions is achieved in a field of view of over 160 × 220 mm with the jet flow centered in the map, for a total of 12,423 vectors per velocity field. Figure 4.3 shows the field of view covered by all four cameras with respect to the calibration plate to give a sense of the linear transform that needs to be made to map the grids back together and compute the three components of velocity.

4.2.7 Choice of the delay $\Delta t$ between systems

The choice of $\Delta t$ is crucial in the finite difference computation of the time derivative of the velocity. If $\Delta t$ is too small, $\Delta u = u(\vec{x}, t + \Delta t) - u(\vec{x}, t)$ will be of the same order of magnitude as the error associated with the PIV measurements themselves. If $\Delta t$ is too large, then the dynamics of the flow at the time scales of interest will not be resolved and the statistical data would be aliased since the
smallest time scales of the flow (the Kolmogorov scales) would not be resolved. An extensive study on that subject was carried out by Perret et al. [86]. Using the time-resolved laser doppler anemometry (LDA) measurements performed by Hall [51] on the same jet at several downstream positions and at a Mach number of 0.6, the root-mean-square value (RMS) of $\Delta u$ can be calculated for different values of $\Delta t$.

Figure 4.4 shows the RMS change in velocity $\Delta u_{rms}$ as $\Delta t$ is increased from 0 to 300 $\mu$s. The plot is shown for several radial positions as they all must be considered in performing cross-plane PIV. This information was used as a guide in
determining an \textit{a priori} reasonable value for $\Delta t$. For small $\Delta t$, the RMS velocity difference should fit a parabola since it is proportional to $\Delta t^2$. Unfortunately, the data does not permit to visualize the evolution of the quantity between $\Delta t = 0$ and $\Delta t = 33$ µs due to the limited sampling frequency of the LDA (30 kHz). On the other hand, it is possible to get an \textit{a posteriori} estimate of the Kolmogorov microscale $\tau$ in the flow to validate the choice of $\Delta t$. An estimate of the Kolmogorov time scale for isotropic homogeneous turbulence is given by [101]:

$$\tau = \left( \frac{\nu}{\epsilon} \right)^{1/2}$$  \hspace{1cm} (4.3)

where $\epsilon = u_{rms}^3/\ell$ is an inviscid estimate of the dissipation rate, $\nu$ is the kinematic viscosity of air, and $\ell$ is the integral scale of turbulence, which will be assumed to be equal to the jet exit diameter $D$ at the downstream position of $x/D = 8$. The maximum RMS value of the fluctuating axial velocity at that location is approximately equal to 30 m/s (about 15\% of the centerline velocity, see Figure 4.17),

Figure 4.4: RMS of the velocity change $\Delta u$ as a function of $\Delta t$, using LDA measurements, at several radial locations, $x/D=6$. 

\[ \begin{align*}
\text{\textcopyright American Institute of Aeronautics and Astronautics, 2006.}
\end{align*} \]
which yields a Kolmogorov time scale in the range of 5-10 $\mu$s for the axial dynamics and a Kolmogorov time scale in the range of 10-15 $\mu$s for the cross-flow dynamics. However, the Eulerian acceleration measurement is fixed in space and the flow is convecting through the plane of measurement while simultaneously changing due to the turbulence. Two consecutive velocity field measurements therefore measure velocity changes in time along with the spatial gradients convecting through the plane. In the present experiment, a $\Delta t$ of 25 $\mu$s was chosen for the downstream positions from $x/D = 3$ to $x/D = 8$ and a $\Delta t$ of 35 $\mu$s for $x/D = 8.5$ to $x/D = 10$ since the time scales in the flow are larger for further downstream positions. These values are a compromise between measuring above noise level derivatives and remaining in the most linear part of the curve shown in Figure 4.4. The smallest time scales in the flow are therefore not fully resolved in the calculation of the acceleration; however these scales are not resolved spatially either since the spatial resolution is 1.8 mm and the Kolmogorov scales are believed to be much smaller at this Reynolds number. The smallest scales are hence filtered out both spatially and in time. However resolving the smallest time scales in the flow is not a concern in this study since the POD is applied to each individual velocity field before taking the time derivative. The dynamics that are intended to be captured are that of the large and intermediate scales in the flow. Figure 4.5 shows the two consecutive measurements of the 3-component velocity field, the out-of-plane component of velocity is in color. A slight distortion and displacement even of the largest structure (at $x=20$ mm and $y=20$ mm) is noticeable. An estimate of the time it takes for this structure to convect through the plane of measurement is given by dividing the size of the structure ($\simeq 20$ mm) by the axial convection velocity (130 m/s), which yields a time of about 150 $\mu$s. It would therefore be expected to see a change in the structure in 25 $\mu$s merely because of the fact that it is convecting through. The changes due to turbulence, as pointed out by a reviewer, indeed scale with the ratio of the size of the structure divided by the standard deviation of the axial velocity which yields a much larger time scale on the order of 500 $\mu$s.
Figure 4.5: Instantaneous 3-component velocity fields measured by both systems with a delay $\Delta t = 25\mu s$.
for the largest structures. The dynamics captured by the Eulerian acceleration measurement account for both convection of the flow and turbulence convecting with the mean flow.

The delay $dt$ between the two frames from each system that are used to compute each velocity field was chosen to be 5 $\mu$s for $x/D = 3$ to $x/D = 8$ and 7 $\mu$s for $x/D = 8.5$ to $x/D = 10$ as the out-of-plane velocities become smaller with downstream position and the particles require more time to travel the same number of pixels (with an average displacement of 5 pixels). This is important since the quality of the PIV cross-correlations depends on how much each particle is displaced in its interrogation area. The delay $\Delta t$ between two consecutive velocity measurements (i.e., between both PIV systems) is therefore in all cases 5 times greater than the delay between two frames used to compute each velocity field.

4.3 Statistics of the M=0.6 jet using PIV

The statistics of the Mach 0.6 axisymmetric jet are presented along with estimates of uncertainties associated with the PIV measurements.

4.3.1 Errors, biases and uncertainty in PIV measurements

Multiple sources of error affect the quality of PIV measurements of velocity and acceleration in fluid flows. Theoretical studies on PIV measurement precision [111] as well as experimental accounts of PIV errors [27] can be found in the literature as it is now a recognized fact that PIV, like all experimental measurement techniques, is not without flaws, therefore special care and precaution must be taken when performing PIV measurements especially in high-speed flows where the errors can become large. All of the errors discussed in this section have been at least addressed and minimized at the best of the author’s ability to constitute an experimental
database of the highest quality possible with the current instrumentation. An estimated global uncertainty in the measurement is given at the end of this section in a table taking into account the contributions of all the different errors related to PIV measurements.

- From a purely statistical point of view, there exists an inherent error in the fact that the number of samples is finite [12]. The estimated mean value of any random quantity $x$ has an associated error of $\sigma_x/(\mu_x\sqrt{N})$ with $\sigma$ the standard deviation at spatial point $x$, $\mu$ the true average value at point $x$ and $N$ the number of samples. The mean square estimates have an associated random error of $\sqrt{2/N}$. Both errors scale as the inverse square root of the number of samples, which means that the number of samples has to be increased four times to halve the error. For this reason, a total of 4000 velocity fields were acquired per PIV system and per downstream position, which amounts to over 120,000 3-component velocity fields and 480,000 images with a total disk space usage of over 1.8 TB (=1800 GB). Each PIV system is run streaming the data directly to two external hard drives of 1 TB each since the camera buffers are not large enough to hold the amount of data. This limits to 1 Hz the maximum rate at which the PIV systems can run continuously. With an amount of $N=4000$ samples per downstream location, the mean value error and the RMS value error are estimated to be respectively equal to 0.5% and 2% for the axial component of velocity; 6% and 2% for the radial component of velocity. The mean azimuthal component of the velocity converges to zero in the whole domain so the associated error cannot be computed. The error is much larger for the radial than the axial component since its standard deviation is much greater than its mean value. A plot of the convergence of the mean values and RMS values at several radial locations in the jet, relative to the best estimate using all samples ($N=4000$) is shown in Figure 4.6. The relative convergence errors
Figure 4.6: Relative convergence error $\xi$ as defined in the text for the axial and radial velocity components, at several radial locations, $x/D=8$.

$\xi_u$ and $\xi_{\sqrt{u'^2}}$ shown in the figures are equal to (with $u_i$ the velocity at time $t_i$):

$$\xi_u = \frac{\frac{1}{n} \sum_{i=1}^{n} u_i - \frac{1}{N} \sum_{i=1}^{N} u_i}{\frac{1}{N} \sum_{i=1}^{N} u_i} \times 100 \quad (4.4)$$

$$\xi_{\sqrt{u'^2}} = \frac{\sqrt{\frac{1}{n} \sum_{i=1}^{n} u_i^2} - \sqrt{\frac{1}{N} \sum_{i=1}^{N} u_i^2}}{\sqrt{\frac{1}{N} \sum_{i=1}^{N} u_i^2}} \times 100 \quad (4.5)$$

These figures indeed show that the error associated with using half the number of samples (2000) would increase by only about 1% for the mean axial velocity and 2% for the RMS axial velocity, but by 15% and 1% respectively
for the mean and RMS radial velocity. This shows the very slow convergence of the secondary cross-flow components of velocity because of the dominance of the axial flow. This again stresses the importance of acquiring a large number of samples when measuring velocity using PIV in a plane where the out-of-plane motion is dominant.

- The second source of error is random noise and is considered to be relatively small overall. It is mainly due to electrical noise in the cameras, cables and boards, uncertainty in time delay generation and sub-pixel correlation peak localization.

- The third source of error is a bias associated with the velocity measurement due to the algorithms used to compute the velocity fields. A thorough report on the uncertainties associated with PIV measurements can be found in the book by Raffel et al. [89]. The PIV images are divided into interrogation areas to compute a local average particle displacement per interrogation area. Pixel-locking is one such bias which is a function of the measured velocity, the particle size relative to the pixel size in the image and the fill ratio of the CCD sensor. The computation of the velocity field from two consecutive images of the seeding particles assumes that the displacement of the particles during this $dt$ is small compared to the displacement of the particles on the smallest time scales of the flow. This assumption holds in the present study since the Kolmogorov time scales were estimated to be twice as large as the $dt$ used to compute the velocity field. However this could be an issue in higher velocity flows, in which case an aliasing error would be associated with the velocity measurement.

- The fourth source of noise is associated with the velocity gradients in the flow, especially when there is a strong mean velocity gradient that makes the particles instantaneously cluster together in the flow and in turn induces errors in the estimated velocity within the interrogation area. Non perfectly
uniform seeding creates noise or inconsistencies in the measurement. Such gradient noise is believed to be significantly responsible for the uncertainty in the measurement in the present flow. The measurement difference shown in Figure 4.9 with $\Delta t$ close to zero is mainly due to this source of error. It has also been observed by Christensen and Adrian [27] in their DT-PIV measurements of a boundary layer flow, especially close to the wall where the gradients of axial velocity are large.

- More importantly, where random errors can be quantified and often reduced using a large enough amount of samples, possible bias errors need to be addressed since they can mislead the interpretation of the results if ignored and cannot be corrected for a posteriori. The most likely bias error in particle image velocimetry is present when taking measurements where the largest velocity component is out-of-plane, which is the case here. The perpendicularity of the mean jet flow to the laser sheets is crucial since the instantaneous axial velocity $u_x$ is about two orders of magnitude greater than the instantaneous in-plane components ($u_r$ and $u_\theta$). Figure 4.7 shows a laser sheet not
orthogonal with the main jet axis. Then the biased measurements are as follows (shown here for the mean velocity but also true instantaneously):

\[
U_{x}^{biased}(r) = U_x(r)\cos(\Psi) + U_r(r)\sin(\Psi)
\]  

\[
U_{r}^{biased}(r) = U_r(r)\cos(\Psi) + U_x(r)\sin(\Psi)
\]

In our case, even a slight non-orthogonality would create a bias in the mean cross-plane velocity field of the same order of magnitude as \(U_r\) and \(U_\theta\). On the other hand, the bias caused to \(U_x\) is minimal relative to its magnitude in the case of a dominant out-of-plane motion. For example, let \(U_x\) at the centerline be equal to 200 m/s, \(U_r\) be equal to 0 m/s and \(\Psi\), the misalignment angle, be equal to 1°. Then the biased measurements at the centerline would be:

\[
U_{x}^{biased}(0) = U_x(0)\cos(\Psi) = 199.97 \text{ m/s}
\]

\[
U_{r}^{biased}(0) = U_x(0)\sin(\Psi) = 3.49 \text{ m/s}
\]

The bias can therefore be quite significant in the in-plane measurements since the average magnitude of the velocities is on the order of 5 m/s. An iterative alignment procedure was implemented to minimize this cross-flow bias, the result of which can be seen in Figures 4.13 to 4.16, where no such bias is noticeable. The zero velocity point is indeed located at the center of the jet.

- Another source of bias is noticeable, making the mean in-plane jet profile look lobed in the horizontal direction. This can be seen in Figure 4.8. The bias is found both in the mean velocity profiles with higher horizontal velocities than vertical ones (in a flow that should be axisymmetric) and also in the RMS value of the fluctuating velocities. The reason for this bias, although unaccounted for in their conference paper, has been recently re-discovered
Figure 4.8: Mean cross-flow \((U_r,U_\theta)\), showing a greater horizontal than vertical component of velocity, \(x/D=8\).

and discussed by Wånström et al. [108] with very similar measurements of the axisymmetric jet cross-stream. The overestimation of the horizontal component of velocity was thoroughly reported by Coudert and Schon [33] in 2001 and also discussed by Willert [112] who named it misalignment error. Indeed the slightest misalignment of the laser plane with the object plane (i.e. the calibration plane) will cause the origin (once the images are dewarped) to be slightly horizontally displaced between the cameras of each system resulting in an artificial horizontal velocity. Communication with the manufacturer of the PIV hardware and software used in the present study (Dantec Dynamics) has been inconclusive and it seems like this type of error is not corrected in their software. A post-processing correction of the data (see Coudert and Schon [33]) would be needed in future work if more accurate measurements of the in-plane motion are needed.
Since two PIV systems are available, a tentative measurement of the difference of the quantities measured simultaneously was performed. Ideally, to measure the average error between the systems, one would build a database of simultaneous measurements with the two PIV systems. This task is however technically difficult to perform since the first PIV system has to trigger the second one and the minimum delay between the two systems is about 2 µs which prevents perfectly simultaneous measurements.

Figure 4.9: Average absolute axial velocity difference between the measurements of PIV 1 and PIV 2 with (a) \( \Delta t = 2 \mu s \) and (b) \( \Delta t = 25.3 \mu s \), \( x/D = 8 \).

A plot of the average absolute axial velocity difference is shown in Figure 4.9 for \( \Delta t = 2 \mu s \) and \( \Delta t = 25.3 \mu s \). From these results, the error in the instantaneous measurement of the axial velocity can be calculated, as well as the error in the computation of the time-derivative of the velocity. The latter has been found to be rather large (close to 20%) but it is argued that the spatial filtering provided by the POD, especially on the low-order modes reduces significantly this error. Figure 4.10 shows contours of the difference in measurement of the mean and standard deviation values of the axial velocity between the two systems. The discrepancy between the two systems is shown to be quite small: (1.5%) in the mean axial velocity measurement and close to 10% in the standard deviation. The error in the standard deviation of the in-plane components of velocity is estimated to be somewhat larger than that of the axial component mainly because of the
Figure 4.10: Contours of (a) \((U_{PIV2} - U_{PIV1})\) and (b) \((\sigma_{u_{PIV2}} - \sigma_{u_{PIV1}})\), in m/s, \(x/D=8\).

“lobed” bias discussed previously. For a more detailed study of the dynamics of the in-plane components (or for CFD data validation), a transfer function can be derived to correct for this bias but it is not thought to be a major concern for the present study.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>(U)</td>
<td>2%</td>
</tr>
<tr>
<td>(V, W)</td>
<td>10%</td>
</tr>
<tr>
<td>(u)</td>
<td>4%</td>
</tr>
<tr>
<td>(v, w)</td>
<td>5%</td>
</tr>
<tr>
<td>(\sigma_u)</td>
<td>10%</td>
</tr>
<tr>
<td>(\sigma_v, \sigma_w)</td>
<td>20%</td>
</tr>
<tr>
<td>(\partial u/\partial t)</td>
<td>20%</td>
</tr>
<tr>
<td>(\partial v/\partial t, \partial w/\partial t)</td>
<td>25%</td>
</tr>
<tr>
<td>(\partial a/\partial t)</td>
<td>15%</td>
</tr>
</tbody>
</table>

Table 4.1: Uncertainty in the experimental measurements of velocity and acceleration using PIV.
The overall uncertainty in the measurement of the different quantities presented in this manuscript is summarized in Table 4.1, all values are valid for the downstream position of x/D=8 and a similar uncertainty investigation will be performed for all downstream positions when the data is used in further developments.

### 4.3.2 Mean statistics

The mean axial velocity profiles at all measured downstream positions (from 3 to 10 diameters) are shown in Figures 4.11 and 4.12. It shows the collapse of the potential core (i.e. where the centerline axial velocity becomes smaller than 99% of the jet exit velocity) to be at around x/D=5. This is consistent with the measurements performed by Tinney [102] in the same facility on the same jet conditions using both a pitot-static probe and also an LDA. The mean axial velocity has a maximum of 200 m/s in the core jet flow and falls to around 3 m/s in the co-flow. The statistics of the mean axial velocity are seen to be nicely axisymmetric in the contour plots (Figure 4.12). The center of the jet within the measurement window was localized using an algorithm that fits an ellipse to each contour and finds the average center of the different ellipses at

![Figure 4.11: Mean axial velocity profiles.](image-url)
Figure 4.12: Mean axial velocity contours.

each downstream position. The average cross-flow velocity fields \((U_r, U_\theta)\) for all downstream positions are shown in Figures 4.13 to 4.16.
Figure 4.13: Mean cross-flow \( (U_r, U_\theta) \) at (top) \( x/D=3 \) and (bottom) \( x/D=4 \)
Figure 4.14: Mean cross-flow \((U_r, U_\theta)\) at (top) \(x/D=5\) and (bottom) \(x/D=6\)
Figure 4.15: Mean cross-flow ($U_r, U_\theta$) at (top) $x/D=7$ and (bottom) $x/D=8$
Figure 4.16: Mean cross-flow ($U_r, U_\theta$) at (top) $x/D=9$ and (bottom) $x/D=10$
The position of the center of the jet, where $U_r=U_\theta=0$ is, as mentioned previously, very sensitive to the perpendicularity of the PIV measurement plane with respect to the true jet axis. The iterative alignment process has shown to be effective at minimizing this type of bias.

A measure of the turbulence intensity, i.e. the ratio of the standard deviation of the velocity normalized by the mean jet exit centerline axial velocity $U_c$, is shown in Figure 4.17 for all three components of velocity $u$, $u_r$ and $u_\theta$. Consistent with the common finding, a maximum turbulence intensity is observed at the center of the shear layer, around 15% at $x/D=3$ it is still 13% at $x/D=10$. The turbulent fluctuations of the radial and azimuthal components of velocity are found to be significantly smaller, ranging from 4% of $U_c$ at $x/D=3$ to between 7% and 8% at $x/D=10$. The increase in turbulence intensity with downstream position for the in-plane components is due to the development of three-dimensional turbulence in the shear layer as it is convected downstream which also explains the decrease in turbulence intensity of the axial component. The turbulence only reaches isotropy far downstream [113]. The slightly higher turbulence intensity in $u_\theta$ is within the uncertainty in the measurement of the in-plane components.

### 4.4 Proper orthogonal decomposition as a low-dimensional tool

#### 4.4.1 Why low-dimensional analysis?

Well-resolved databases of highly turbulent flow fields are now made available, whether it is through experiments (in particular using PIV), or through computational fluid dynamics: direct numerical simulation (DNS) and large eddy simulation (LES) being more and more widely used with the increase in computational capability. To name only a few of the techniques used in the fluid dynamics com-
Figure 4.17: Turbulence intensity for velocity components $u$ (top), $u_r$ (middle) and $u_\theta$ (bottom).
munity to make physical sense of the information, Fourier analysis, wavelet analysis and proper orthogonal decomposition (POD) are certainly the most commonly utilized. There are several critical reasons why one needs to rely on low-dimensional mathematical analysis when dealing with complex fluid flows:

- The instantaneous amount of information contained in high Reynolds number turbulent flows is overwhelming. This is due to the nonlinear nature of the Navier-Stokes equations that govern fluid flows and the challenging nature of these equations explains why turbulence is a problem that has not yet been solved. However if one recognizes the presence of a certain amount of coherence and large-scale organization within the chaotic appearance of such flows, it is then possible to focus only on those events and mechanisms of interest using appropriate mathematical tools to “filter-out” the less coherent (or less relevant) part of the information.

- Each mathematical tool has its own formulation but all have the potential of rearranging the information contained in the flow in a certain manner. Fourier analysis splits the information into individual frequencies, wavelet analysis provides the frequency content of the signal for all times and POD supplies a basis that is optimized in terms of energy content. It is hence possible to break down the physics of interest in the flow, gain understanding of the micro-scale mechanisms and explain the macro-scale phenomena observed (e.g. large vortex break down, aerodynamic noise, local surface heating due to turbulence, flow separation, cavity modes...).

- When the aim is to control a flow in real-time using closed-loop control, the information provided by the multiple sensing devices has to be acquired, analyzed and an appropriate signal must be fed back to the actuating devices to accomplish the control objective at all times and this operation needs to be performed on the same time scale as the dynamics of the flow targeted to be controlled. Is is therefore imperative to treat only the part of the
information provided by the sensors that is crucial to performing the control and nothing more. The less information necessary to accurately describe the flow state, the faster the control will be. It is therefore a compromise between accuracy of the state estimate and speed of control. Therein lies the importance of the efficiency of the low-dimensional techniques to represent the flow state in only few modes. This capability can also be compensated for by using advanced control techniques (e.g. Kalman filters).

The technique that has been selected to accomplish the task of rendering a low-order representation of the flow is the POD since it presents many practical mathematical advantages and is optimally efficient in breaking down the information in terms of energy.

4.4.2 The Classical POD

The mathematical background at the origin of the POD was laid out very early since it is known in other disciplines under different names. The Karhunen-Loève decomposition [62] [69] is the theoretical counterpart of the POD which is typically applied empirically. Principal component analysis (PCA) is another name relating to the Karhunen-Loève and it is used in general to reduce the dimensions of complex databases in a very broad range of applications from data compression to image/video processing. POD also relates to Fourier analysis and can be considered a generalization of Fourier series in the non-homogeneous and non-periodic directions of the flow. As an example, the Fourier transform is often combined with the POD in the axisymmetric jet flow when strict homogeneity and periodicity are assumed to be true in the azimuthal direction. POD is then applied in the radial (inhomogeneous) direction. The theory of the original version of the proper orthogonal decomposition was introduced in 1967 to the fluid dynamics community by John Lumley [70] and its use has since spread to a wide range of experimental and computational applications. Lumley presented the technique as
an unconditional and unbiased way to extract coherent structures from a flow since it needs no *a priori* knowledge of the flow field. A coherent structure in Lumley’s sense is defined as a spatial structure that has the highest projection on average onto the different flow realizations of the database. It is also important to note that the relative amplitude of the identified structure is of no interest, it is only the strength of the projection on the realization, or their degree of parallelism that matters. The basis $\phi^{(n)}$ representing the spatial structures is therefore normalized and the problem becomes the following constrained optimization expression:

$$\max_{\varphi} \frac{\langle |\langle \bar{u}, \varphi \rangle |^2 \rangle}{\| \varphi^2 \|} = \frac{\langle |\langle \bar{u}, \phi \rangle |^2 \rangle}{\| \phi^2 \|} \quad \| \phi \|^2 = 1$$  \hspace{1cm} (4.10)

where the inner product is defined, along with the corresponding norm $||.||$ in the Hilbert space of square-summable functions $L^2$, using $\vec{f}$ and $\vec{g}$ two functions of this space with components $(f_1, f_2, f_3)$ and $(g_1, g_2, g_3)$ respectively, as:

$$\langle \vec{f}, \vec{g} \rangle = \int_D \left( f_1 g_1 + f_2 g_2 + f_3 g_3 \right) d\vec{x}$$  \hspace{1cm} (4.11)

It can be easily shown that the problem is equivalent to solving the following Fredholm integral eigenvalue problem:

$$\int_D R_{ij}(\vec{x}, \vec{x}') \phi_i^{(n)}(\vec{x}') d\vec{x}' = \lambda^{(n)} \phi_i^{(n)}(\vec{x}), \quad n = 1, 2, 3, \ldots$$  \hspace{1cm} (4.12)

where $\lambda^{(n)}$ are the eigenvalues and $\phi_i^{(n)}$ are the eigenfunctions. $R_{ij}(\vec{x}, \vec{x}')$ is defined in the “classical” POD as the ensemble averaged two-point spatial velocity correlation tensor as:

$$R_{ij}(\vec{x}, \vec{x}') = \langle u_i(\vec{x}, t) u_j(\vec{x}', t) \rangle$$  \hspace{1cm} (4.13)

where $u_i(\vec{x}, t)$ is the fluctuating velocity field as defined by the Reynolds decom-
position commonly used in turbulence:

\[ u_i(x, t) = \tilde{u}_i(x, t) - U_i(x) \]  \hspace{1cm} (4.14)

where \( \tilde{u}_i(x, t) \) is the instantaneous velocity field and \( U_i(x) \) the mean velocity field, ensemble averaged over all the realizations of the flow. The POD is in general applied to the fluctuating velocity field to be able to separate the turbulent dynamics from the effect of the mean convection. In the present case the mean flow dominates the axial component of velocity, being an order of magnitude higher than the axial fluctuations, which justifies the need to separate the mean from the fluctuating velocity field. It has however been shown that when applying the POD on the instantaneous velocity field \([32]\), since the mean flow often contains most of the energy, the first POD mode represents the mean flow itself and the next modes are equivalent to that obtained when applying the POD on the fluctuating velocity field.

The domain of integration \( D \) being bounded and \( R_{ij} \) being symmetric, the Hilbert-Schmidt theory applies, ensuring that an infinite denumerable set of solutions to the Fredholm integral problem exists and the following properties are verified:

The eigenfunctions are by construction orthogonal to each other but for convenience are also chosen to be orthonormal, we therefore have:

\[ (\phi^{(m)}, \phi^{(n)}) = \delta_{mn} \]  \hspace{1cm} (4.15)

The eigenvalues are defined positive and are ordered with the first eigenvalue having the highest amplitude and so on:

\[ \lambda^{(1)} \geq \lambda^{(2)} \geq \ldots \geq \lambda^{(\infty)} \geq 0 \]  \hspace{1cm} (4.16)

The time dependent POD expansion coefficient \( a_n(t) \) is obtained by projecting
the flow realizations onto the eigenfunction basis as follows:

\[ a_n(t) = \int_D u_i(\vec{x}, t) \phi_i^{(n)}(\vec{x}) d\vec{x} \]  

(4.17)

It can be shown that these coefficients are non correlated and that their average square value is equal to the corresponding eigenvalue:

\[ \langle a_m(t)a_n(t) \rangle = \lambda^{(m)} \delta_{mn} \]  

(4.18)

Each velocity field realization can be reconstructed using the expansion coefficients and eigenfunctions:

\[ u_i(\vec{x}, t) = \sum_{n=1}^{\infty} a_n(t)\phi_i^{(n)}(\vec{x}) \]  

(4.19)

Also, using Equations 4.19, 4.15 and 4.18 it can be shown that the total turbulent kinetic energy in the flow is represented by the sum of the eigenvalues:

\[ \frac{1}{2} \int_D \langle u_i(\vec{x}, t)u_i(\vec{x}, t) \rangle d\vec{x} = \sum_{n=1}^{\infty} \lambda^{(n)} \]  

(4.20)

This result shows that the magnitude of each eigenvalue \( \lambda^{(n)} \) represents the contribution of its corresponding normalized structure \( \phi^{(n)} \) to the total turbulent kinetic energy and that each structure contributes independently to amount to the total energy. The POD technique is therefore the optimal decomposition of the flow realizations in terms of energy content per structure (or mode), which means that it is not possible to find another orthogonal basis that contains as much turbulent kinetic energy in a given number of modes.

In practice, a low-dimensional representation of a particular velocity snapshot at time \( t_o \) is given by truncating Equation 4.19 to a finite number of modes \( N \):
\[ u_i(\vec{x}, t_o) = \sum_{n=1}^{N} a_n(t_o) \phi_i^{(n)}(\vec{x}) \]  (4.21)

It is also important to note that in a finite measurement domain, the size of the eigenvalue problem is equal to the size of its kernel \( R_{ij} \) which is given by 

\[ N_{POD} = n_c \times N_{grid} \]

with \( n_c \) the number of components of velocity and \( N_{grid} \) the number of spatial grid points of the measurement area. Commonly, a small number of modes (relative to the total number of modes) is sufficient to retrieve most of the turbulent kinetic energy at a given time. The ability of the POD to capture most of the energy in a few modes is very much tied to the type of flow in consideration. A low Reynolds number flow, with large scale structures and significant periodicity will only need the first few modes to retrieve typically more than 95\% of the energy. The higher the Reynolds number, turbulent intensity and intermittency, the greater number of modes will be required to retrieve the same amount of energy, as is the case in the present study.

### 4.4.3 The Snapshot POD

The snapshot version of the POD was introduced by Sirovich in 1987 [96], who showed the mathematical equivalence of performing time averaging or spatial averaging to compute the statistical properties of the flow. The main difference between this method and the classical one is that the dimension of the problem is now reduced to the number of independent snapshots \( N_t \) instead of \( n_c \times N_{grid} \).

These values can be significantly different and the choice of using a particular method will therefore be dependant on the spatial and temporal resolutions of the measurement. Minimizing the numerical size of the problem can be crucial even with the best computing capabilities. Indeed, computational methods such as LES and DNS usually possess a very high spatial resolution and a rather small time history due to computing costs. Likewise, PIV based techniques reach very good spatial resolutions and the POD kernel can quickly become unpracticable. The
snapshots method of the POD is in these cases usually applied. As an example, the present DT-PIV database comprises 4000 snapshots of velocity fields per system and per downstream position with a spatial resolution of 1.8 mm, resulting in about 12,000 3-component velocity vectors per field, a classical POD kernel would therefore generate around 36,000 modes while the snapshot POD kernel would have a rank of 4000. On the other hand, coarser measurements such as provided by rakes of hot-wires typically have a much better time than spatial resolution. The POD kernel in this case converges much faster using the classical method and the size of the problem is much smaller.

In the snapshot POD, the integral eigenvalue problem from Equation 4.12 is shown to be equivalent to:

$$\int_T C(t, t') a_n(t') dt' = \lambda^{(n)} a_n(t)$$

(4.22)

where \(a_n(t)\) are now the temporal eigenfunctions of the problem and \(C(t, t')\) is the two-time correlation tensor defined as:

$$C(t, t') = \frac{1}{T} \int_D u_i(\vec{x}, t) u_i(\vec{x}, t') d\vec{x}$$

(4.23)

For reasons of consistency with the ‘classical’ POD, the latter are arbitrarily chosen to satisfy the following relation:

$$\langle a_m(t) a_n(t) \rangle = \lambda^{(m)} \delta_{mn}$$

(4.24)

The spatial eigenfunctions \(\phi_i^{(n)}(\vec{x})\) must then be defined as:

$$\phi_i^{(n)}(\vec{x}) = \frac{1}{T, \lambda^{(n)}} \int_T a_n(t) u_i(\vec{x}, t) dt$$

(4.25)

so that they remain orthonormal as in the classical POD and verify:

$$\int_D \phi_i^{(m)}(\vec{x}) \phi_i^{(n)}(\vec{x}) d\vec{x} = \delta_{mn}$$

(4.26)
In the same manner, the low-dimensional velocity fields are then reconstructed by projecting the temporal eigenfunctions $a_n(t)$ onto the spatial eigenfunctions $\phi_i^{(n)}(\vec{x})$ using Equation 4.21.

### 4.4.4 A brief POD review

The present paragraph is not intended to provide an exhaustive review of the applications of POD in fluid mechanics but rather to give a brief overview of the work that is relevant to the present topic and that has been critical in making POD the widespread tool it is today. The reader is referred to the book by Holmes et al. [54] or the review by Berkooz et al. [14] for a thorough background on the POD method and its application to dynamical systems. It took a certain time, since its introduction in 1967 by John Lumley [70] in this field, for the method to be applied to experimental flow data. In 1987, Glauser et al. [46] applied the POD to experimental data from cross-wire measurements of the turbulent axisymmetric mixing layer. They were able to show the presence of a coherent “ring-like” structure near the potential-core region of the flow and its breakdown into higher azimuthal modes within the mixing layer. This work confirmed the vision of Lumley that this tool would be able to unconditionally extract coherent information from turbulent flows. The use of POD to study the dynamics of the axisymmetric jet is extensive, much of it originating from the Turbulence Research Laboratory at SUNY Buffalo (now at Chalmers University of Technology) with the work by Citriniti and George [28] investigating the near-field jet with a rake of 138 hot-wire probes, this work being then extended to multiple downstream positions from the near-field with Jung [59] to the far-field with Gamard [42]. When Sirovich introduced the snapshot POD variant in 1987 [96], the method could then be easily applied to highly resolved CFD data which led to the widespread use of POD in the fluid dynamics field. The interest in the low-dimensionality
provided by the POD method gave rise to many studies focused towards developing dynamical systems, with the first successful implementation by Aubry et al. [7] who were able to correctly predict the dynamics of streamwise vortex pairs in a turbulent boundary layer. Zheng [115] and then Ukeiley et al. [106] applied the same idea to predict the dynamics of the mixing layer using a Galerkin projection of the Navier-Stokes equations onto a POD basis and thereby solving the resulting ordinary differential equation numerically. The use of Galerkin projections onto POD bases is discussed in Rempfer [91] and the work by Noack et al. [80] on the control of the laminar cylinder wake using POD reduced order models (POD-ROM) has shown very successful results. POD is now the subject of optimization of reduced order models for performing optimal control [13]. The implementation of low-order dynamical systems using POD to high Reynolds number turbulent flow control applications is yet to come and the present work will lay the first steps towards this challenging goal. In the flow control applications, POD was first used by Glauser et al. [45] (see also Pinier et al. [88]) in a proportional closed-loop approach without the use of a dynamical controller to provide low-dimensional information about the flow state and actuate accordingly. They showed that even in this turbulent flow separation problem, the first POD mode alone was capable of detecting incipient separation and prevent massive separation from occurring. All the details pertaining to this experiment and the development of a dynamical system to improve the applied flow control scheme can be found in Ausseur [8].

4.5 POD results

For reasons discussed in Section 4.4.3, the snapshot version of the POD was chosen to be applied to the databases of velocity fields provided by both PIV systems. Each system provided $N_t = 4000$ independent velocity fields at each downstream position resulting in a total number of modes $N_{POD} = 4000$, instead of over 36,000 ($=n_c \times n_{grid}$) should the classical POD method be applied. Unlike most POD
studies previously conducted on the axisymmetric jet, periodicity and homogeneity of the flow in the azimuthal direction are not \textit{a priori} assumed in the present work. Therefore decomposition into azimuthal Fourier modes is not performed. Instead, a vector implementation of the POD is performed in both azimuthal and radial directions on all 3 components of velocity in the original cartesian measurement grid. The reasons for this choice are multiple:

- PIV data being taken on a rectangular cartesian grid, periodicity in the azimuthal direction is not embedded in the measurement, unlike the measurements of Citriniti [28] where the hot-wire probe rake was designed to measure velocity on an azimuthal grid centered on the jet centerline, in which configuration it is natural to perform the azimuthal Fourier decomposition immediately. In the case of Glauser [44], azimuthal periodicity and homogeneity assumptions had to be made from the start since computing and measurement capabilities were limited and the measurements were made simultaneously along two radial rakes of hot-wires. In the present database, the whole plane is measured simultaneously on a cartesian grid which alleviates the need for a priori assumptions.

- The main goal of this research not being directly the investigation of the azimuthal organization of the jet flow but rather the development of a dynamical system that could then be used for this kind of investigation, such assumptions are not necessary and may in fact artificially force the dynamical system to an axisymmetric behavior if the higher azimuthal modes are truncated.

- The truncation of the dynamical system to $N_{LODS}$ is crucial and care must be taken to insure that no important dynamics are truncated which often results in a dynamical system diverging in amplitude or artificially reaching a non physical limit cycle. Therefore a study of the spatial characteristics of each
individual mode is carried out to understand how each one (axisymmetric or not) contributes to the overall dynamics of the low order system.

- The Fourier azimuthal transform splits the energy spectrum of the eigenvalues into modes characterized by a certain number of periodic “lobes”, but the overall energy contained in each mode could be stemming from a more complex dominant structure that contains aspects of several azimuthal modes, or from several different coherent structures all containing (as it will be shown to be true) aspects of one azimuthal mode.

- In the case where the flow weren’t perfectly axisymmetric (which is always to some extent the case) or perfectly centered on the interpolated polar grid (which in experiments is a challenge), the assumption only holds to within a certain degree of error and a physical interpretation of the results could be misleading when one looks at subtle magnitude differences between azimuthal modes.

### 4.5.1 Convergence of the POD modes

As described in Equation 4.20, the total turbulent kinetic energy (TKE) is represented by the sum of the eigenvalues $\lambda^{(n)}$. It is therefore possible to visualize the contribution of each mode to the TKE which can be insightful as to how much energy is truncated in the low order dynamical system. The individual contribution of each eigenvalue is measured by:

$$\Lambda^{(n)} = \frac{\lambda^{(n)}}{\mathcal{E}} \times 100$$

(4.27)

with $\mathcal{E}$ the total TKE defined as:

$$\mathcal{E} = \sum_{n=1}^{N_{POD}} \lambda^{(n)}$$

(4.28)

The cumulative and individual contributions of each mode are shown in Fig-
Figure 4.18: Cumulative and individual convergence of the POD eigenvalues. Unlike what is commonly found in low Reynolds flows that exhibit large scale structures or organized shedding of vortices, the first mode alone here only contains 3.5% of the TKE, but the convergence of the POD is quite surprisingly steep (relative to the total number of modes) and only 12 modes are necessary to build back 30% of the TKE, 34 modes for 50%, 88 modes for 70% and to retrieve 90% of the TKE 305 modes are required out of the 4000 total. These figures are important since they indicate how much of the TKE is truncated in the low order dynamical systems. This truncation greatly impacts how the system behaves in its short time prediction as discussed in Section 4.6. It is however attractive to be able to reduce the dimensions of the problem by two orders of magnitude but still retaining over 50% of the TKE. This feature is crucial for the feasibility of low order dynamical systems. Examining the individual energy content of each eigenvalue relative to the total energy in Figure 4.19, it is noticeable that some of the first modes have similar levels of energy before a steep drop leads to another couple of modes with similar energy content and so on. This kind of mode-grouping is typically found in flows with strong convective eddies, where the modes are usually grouped in pairs with both modes in each pair very similar in spatial distribution but out-of-phase by $\pi/2$. The following section examines the spatial distribution of the structures identified by the POD eigenfunctions.
4.5.2 POD eigenfunctions and structure identification

In an effort to find a mathematical and objective way of describing coherent structures in turbulence, Lumley [70] found that the POD constitutes a way to exhibit a complete basis of functions that best represents the flow realizations in a given database, with the first mode being the most representative. In other words, the first POD eigenfunction is the vector that possesses the highest average projection on each one of the realizations (in the sense of energy), it is the most parallel on average to the database. Each eigenfunction can therefore be interpreted as a “structure” that occurs in the flow more often than its subsequent eigenfunctions. It is however still not clear within the fluid dynamics community (and therein lies the controversy) whether or not energy is the right quantity to characterize actual physical structures. Therefore in the present text, the term structure, when used in the context of the POD is not intended to carry the meaning of a physical structure, as one could imagine it, but rather a more broad sense of the term, as meant by Lumley or defined by Hussain [56], being an event “most likely” to occur in the flow.

Figure 4.20 shows the spatial modes $\phi_n^{(n)}$ for POD modes 1 to 6. It is in-
teresting to notice that the 2D vector POD exhibits structures in the low order modes that look very much like ones coming from a Fourier azimuthal decomposition. This is however not surprising since it can be shown [13] that applying a Fourier decomposition in a homogeneous and periodic direction (the azimuthal direction in the axisymmetric jet) is equivalent to applying the POD in that direction. It is nonetheless insightful in a first approach to let the vector implementation of the POD pull out the most azimuthally energetic structures. One can recognize that the first POD mode resembles a Fourier mode-2-like structure with two positive and two negative lobes which would suggest that this type of low-dimensional structure is most likely at this downstream position. The next three POD modes (2, 3 and 4) also resemble an azimuthal mode 2 but somewhat asymmetric and each in a different phase. Following a fruitful discussion with Dr. Jonathan Naughton [78], a combination of all three modes can recover the structure from POD mode 1 out-of-phase by $\pi/2$. The combination of the two making non-swirling mode 2 the dominant structure in the flow at this downstream position. Figure 4.24 shows the structure recovered by performing a spatial average over POD modes 2, 3 and 4. POD modes 5 and 6 approach a Fourier mode 3 pair of sines and cosines (i.e. out-of-phase by $\pi/2$). In Figure 4.21, one can easily recognize a Fourier mode-4-like pair (POD modes 7 and 8) and the same as with Fourier mode 2 occurs with Fourier mode 5: Figure 4.25 displays (on the right) the combination of POD modes 9, 11 and 12 that results in the $\pi/2$ out-of-phase representation of the Fourier mode-5-like, its counterpart being represented in POD mode 10 (on the left). Likewise, Figure 4.22 shows that POD modes 18, 19 and 20 represent a Fourier mode-6-like structure but it is interesting to notice that more complex patterns contain more energy than the latter structure, meaning that

the flow is not solely dominated by azimuthally periodic structures and an azimuthal Fourier decomposition representation makes this feature difficult to grasp. Indeed POD modes 14, 15 and 16 reveal a structure containing an “inner” Fourier mode-3-like structure surrounded by an “outer” Fourier mode-3-like struc-
Figure 4.20: Eigenfunctions of the axial velocity component $\phi_u^{(n)}$, modes $n=1$ to $n=6$. 
Figure 4.21: Eigenfunctions of the axial velocity component $\phi_u^{(n)}$, modes $n=7$ to $n=12$. 
Figure 4.22: Eigenfunctions of the axial velocity component $\phi_{u}^{(n)}$, modes n=14, 15, 16, 18, 19 and 20.
Figure 4.23: Eigenfunctions of the axial velocity component $\phi^{(n)}_u$, modes $n=13, 17, 21, 22, 23$ and $24$. 
Figure 4.24: Eigenfunctions of the axial velocity component, (left) $\phi_{u}^{(1)}$ and (right) $(\phi_{u}^{(2)} + \phi_{u}^{(3)} + \phi_{u}^{(4)})/3$.

Figure 4.25: Eigenfunctions of the axial velocity component, (left) $\phi_{u}^{(10)}$ and (right) $(\phi_{u}^{(9)} + \phi_{u}^{(11)} + \phi_{u}^{(12)})/3$.

ture. Similarly POD modes 21 and 22 display inner and outer Fourier mode-4-like structures which are identified by the red circles in Figure 4.26. The interpretation of these patterns as being actual physical structures that occur in the flow can be quite controversial. Nonetheless it is mathematically sound and not physically far-fetched to think of these structures as the most statistically probable energy containing structures. Since each one of these structures $\phi^{(n)}$ is weighted by the
corresponding POD expansion coefficients to rebuild the instantaneous snapshot (see Equation 4.21), one can imagine cases where one of the structures would be weighted much more importantly than the other ones to constitute a velocity field very much similar to the particular eigenfunction.

Furthermore, the observation of the eigenfunctions’ spatial structure has revealed crucial elements for the development of the dynamical system as is discussed in the next section. Indeed it is clear that the eigenfunctions can be grouped by 2, 3 or 4, as was shown previously according to structure similarity. Therefore care needs to be taken when truncating the system at a certain level since taking only part of a group of structures could result in its inability to correctly represent an individual event. The different levels of truncation will be discussed in the Section 4.6.

4.5.3 POD velocity field reconstructions

To give a sense of the instantaneous high dimensionality of the flow and to show the ability of the POD to capture the main features in a low number of modes,
Figure 4.27 shows the axial component of velocity at three independent times measured by the PIV and its reconstruction using only 100 POD modes out of 4000 (2.5%). The mean axial component of velocity is added to the fluctuating

reconstructions in Figure 4.28 thus retrieving the high-speed core region which has
at this stage already collapsed and the distortion of the outer shear layer as the large-scale structures are convected through. Figure 4.29 shows the in-plane com-

Figure 4.28: Identical plots as in Figure 4.27 with the mean axial velocity field added, color scale from 0 to +190 m/s.

ponents of velocity at the same instantaneous times, here again showing the small scales of the cross-flow plane dynamics and the high intensity of turbulence. For the in-plane components of velocity, the mean values are much smaller the fluc-
Figure 4.29: In-plane fluctuating velocity components of (left) the original instantaneous PIV velocity field and (right) the 100 POD mode reconstruction, at three independent times (t=236, 238 and 249), at x/D=8.

fluctuating values so adding the mean flow only slightly modifies the instantaneous representation of the flow. The time scales of the dynamics retrieved in the low order dynamical system is very dependent upon the level of truncation (i.e. the number of modes kept to train the dynamical system). Therefore it is shown in
Figure 4.30: (a) Axial fluctuating velocity component of the original instantaneous PIV velocity field, and POD reconstructions using (b) 12 modes (30% of TKE), (c) 34 modes (50% of TKE), (d) 88 modes (70% of TKE), (e) 305 modes (90% of TKE), and (f) 4000 modes (100% of TKE), time t=454, x/D=8, color scale from -60 to +60 m/s.
Figure 4.30 how truncating at different mode numbers affects the accuracy of the reconstruction. Obviously, a compromise must be made between the amount of energy retained in the system and the size of the system that will be built. The latter is greatly limited by the fact that inverting a large size matrix often results in it being ill-conditioned, which introduces a significant amount of numerical error in the system and prevents the dynamical system from staying bounded even for short times. It is acknowledged that a 12-mode representation of this particular snapshot is only a very rough estimate but that a 34-mode reconstruction with 50% of TKE is much more efficient and can provide a nice compromise between low-dimensionality and accuracy.

4.6 Low-order dynamical system development

Turbulence and the transition process from laminar to turbulent flow make the prediction of flow behavior extremely challenging. The dimensions and complexity of the problems increase exponentially when turbulence appears. Even though the characteristics of turbulent flows seem to be chaotic and random in nature, it is possible to identify organized and reoccurring features in these types of flows [20]. It is therefore useful to perform low-dimensional analysis on highly unsteady flows that appear complex at a first glance but that hide intelligible information. The knowledge of such information can be a key to understanding the underlying physical mechanisms which can thereby guide flow control strategies. Given a complex system fluid flow, the goal in developing reduced order models (ROM) is to solve only a part of the infinite dimensional governing evolution equations (partial differential equations) by projecting these equations onto a subspace of finite dimension made up of a reduced basis of empirical eigenfunctions (like those provided by the POD). The level of complexity of a fluid flow increases with Reynolds number, but experiments at many different Reynolds numbers have shown that low-order
analysis is effective throughout the whole range and that organized structures exist at all Reynolds numbers. Several techniques enable the construction of a low-order model in fluid dynamics. The most widely used is the POD-Galerkin method that involves projecting the Navier-Stokes equations onto a truncated orthogonal POD basis, leaving only a small number of modes to solve for. The initial system of partial differential equations (the Navier-Stokes equations) is now a system of ordinary differential equations (ODE) that can be solved using mathematical methods.

In the area of fluid dynamics, the groundbreaking work of Aubry et al. [7] was the first to show predictive capability in a complex fluid flow. They were able to correctly model the dynamics of the coherent structures inside a boundary layer. Much background on reduced order models (ROM) can be found in Holmes et al. [54]. The other main motivation behind developing low-order dynamical systems (LODS) is for closed-loop control. Having a LODS is a means of predicting the low-dimensional state of the flow at an instant later in a very cost effective manner (i.e. computationally very fast). Indeed, since in most flow control applications only selected features in the flow need to be known and predicted (e.g. flow separation, laminar/turbulent flow, coherent structure identification, aero-acoustic noise level...), a truncated representation of the flow very often suffices to make the decision on the control input to the flow. The only drawback to such approaches is that a priori measurements (or simulations) and quite heavy data post-processing need to be performed to develop the dynamical system off-line. The main advantage is that the predictive scheme is quite simple computationally and can be easily implemented online, i.e. in real-time, even in high-speed applications.
4.6.1 POD-Galerkin inspired Lods

A Galerkin projection is the projection of a system of partial differential equations onto a finite basis of linearly independent functions. This projection can then be truncated to model only a part of the system. The proper orthogonal decomposition is a method that conveniently complements the Galerkin projection since the POD basis is orthonormal by construction and the functions are optimized in terms of the square of the velocity (i.e. the energy), hence when truncating the system at a low mode number, only the most energetic part of the flow is modeled in the system. This, as will be seen, can also present drawbacks. In low Reynolds number laminar flows, the POD captures between 90 and 100% of the energy in the first couple of modes, in an intermediate Reynolds number range ($\sim 10^4$), between 50 and 70% of the energy can be captured with the first couple of modes and at high Reynolds number turbulent flows, the first modes only capture on the order of 20% of the TKE. This amount of energy might however be sufficient to capture the interesting features of the flow. Since turbulence intensities are high in such flows, a large amount of information can be neglected to extract only the coherent structures from the flow. However, the neglected part of the flow often contains the smallest scales which are responsible for the energy dissipation. This is the major reason for which, especially in high Reynolds number applications, the dynamical systems diverge very easily and only short time predictions are possible. There are several ways to compensate for the truncation of the dissipative scales when developing a dynamical system. These methods are discussed in the following paragraphs. Since an empirical method is used in the present work and the full Galerkin projection with explicit calculations of the projection coefficients is not performed, the analysis tied to the Galerkin projection in the case of incompressible flows is only summarized in the following equations but a more thorough analysis can be found in [13].

In non-dimensional form, the incompressible Navier-Stokes equation can be
written as follows:

\[
\frac{\partial \vec{u}}{\partial t} + (\vec{u} \nabla) \vec{u} = -\nabla p + \frac{1}{Re} \nabla^2 \vec{u}
\]  

(4.29)

with the inertial terms on the left-hand-side balanced by the pressure gradients and the viscous forces. The POD-Galerkin method consists in projecting these equations onto a POD basis \( \phi^{(n)} \) as defined in Equation 4.10 using the \( L^2 \) inner product:

\[
\left( \phi^{(n)}, \frac{\partial \vec{u}}{\partial t} + (\vec{u} \nabla) \vec{u} \right) = \left( \phi^{(n)}, -\nabla p + \frac{1}{Re} \nabla^2 \vec{u} \right)
\]  

(4.30)

Using Equation 4.17 we have:

\[
\frac{\partial \vec{u}}{\partial t} = \frac{\partial}{\partial t} \sum_{m=1}^{N_{trunc}} a_m \phi^{(m)} = \sum_{m=1}^{N_{trunc}} \phi^{(m)} \frac{\partial a_m}{\partial t}
\]  

(4.31)

and since the basis \( \phi^{(n)} \) is orthonormal, \((\phi^{(m)}, \phi^{(n)}) = \delta_{mn}\) and:

\[
\left( \phi^{(n)}, \frac{\partial \vec{u}}{\partial t} \right) = \left( \frac{\partial a_n}{\partial t} \right)
\]  

(4.32)

and therefore 4.30 is equivalent to:

\[
\frac{d a_n}{dt} = - (\phi^{(n)}, \nabla p) + \frac{1}{Re} \left( \phi^{(n)}, \sum_{m=1}^{N_{LODS}} a_m \nabla^2 \phi^{(m)} \right)
\]

- \( \left( \phi^{(n)}, \sum_{m=1}^{N_{LODS}} \sum_{k=1}^{N_{LODS}} a_m a_k (\phi^{(m)} \nabla) \phi^{(k)} \right) \)  

(4.33)

Unless the pressure field is available (when using CFD for example) in which case this term can be calculated directly, one needs to rely on assumptions on the boundary conditions of the problem. In an open flow like the jet flow, we can safely assume that the eigenfunctions are homogeneous (i.e. are equal to zero) in the outer region so that, using an integration by parts and rigorous analytical
manipulations [106], the pressure gradient term vanishes. In other cases like the boundary layer dynamical system of Aubry et al., this term needed to be modeled since the above assumptions are not valid. Rempfer [90] also showed that by taking the curl of the POD eigenfunctions, one could build a basis tied to vorticity on which to perform the projection. In that case, there is no pressure term that needs to be modeled for the Galerkin projection.

The equations resulting from the projection in Equation 4.33 are as can be seen quadratic (as are the Navier-Stokes equations) but an important point lies in the fact that the Reynolds decomposition is used in the present case. Indeed, when separating the mean flow from the fluctuations in the projections, extra terms describing the interaction between the mean velocity and the turbulent fluctuations are derived [106]. Since these terms are cubic in nature, an additional cubic term will be added to the modeling ODE to model this interaction. The second motivation for adding a cubic term in our case is that the flow cannot be considered incompressible, even though the previous developments were made using the incompressible version of the Navier-Stokes equations. Rowley et al. [93] apply a Galerkin projection to the full compressible Navier-Stokes equations and introduce a more appropriate inner product adapted for compressible flows. They also show that since the derived compressible equations are this time cubic in nature, a cubic model needs to be considered. Therefore, the following general ODE will be used in the development of an empirical low order dynamical system of the jet, as was similarly used by Perret [85]:

\[
\frac{da_n}{dt} = D_n + \sum_{j=1}^{N_{\text{LODS}}} L_{nj}a_j + \sum_{j=1}^{N_{\text{LODS}}} \sum_{k=1}^{N_{\text{LODS}}} Q_{njk}a_ja_k + \sum_{j=1}^{N_{\text{LODS}}} \sum_{k=1}^{N_{\text{LODS}}} \sum_{l=1}^{N_{\text{LODS}}} C_{njk\ell}a_ja_ka_{\ell} \quad (4.34)
\]

where \(D_n, L_{nj}, Q_{njk}\) and \(C_{njk\ell}\) are the constant, linear, quadratic and cubic terms of the dynamical system that need to be identified. As described above, these can
be calculated directly using the analytical expressions derived from the Galerkin projection. Very finely resolved data (both in time and space) is however needed for such calculations and this direct technique would not be appropriate in the case of experimental PIV data where a significant amount of noise would be introduced by the double spatial derivatives and the computation of higher order terms. These coefficients will here be identified using an empirical method commonly referred to as the “moments” method developed at the Laboratoire d’Etudes Aérodynamiques at the University of Poitiers, France [16] [92]; this method is described in the following section.

4.6.2 System identification

The moments method has been described and successfully applied several times [92] [17] [85] and has shown very encouraging results in cases where the flow structures exhibit clear periodicity and a coherent convective behavior. The experimental and computational data used to develop the systems contained this information and very satisfactory predictions were established. The present work extends this technique and its application to a much higher speed flow to see how a more turbulent and less “coherent” database affects the prediction capability of this method. In the present cross-flow measurements, convection is through the plane of measurement and not (as it is often the case) within the plane. Therefore, structures are not convecting through the window and dynamics are thus considerably more difficult to grasp. It is also expected that the truncation effect will have more impact on the system since all the dissipative scales are filtered out of the model. In such a flow, only a maximum of half of the TKE is kept (with 30 modes), the other half being truncated. The system identification method is presented here for completeness but it can also be found in the aforementioned references.
samples of the measurement and not a time-resolved signal. The method in fact converges faster when trained with independent samples rather than with slightly correlated measurements. The input database comprises for each independent time \( t_i \) a pair \((a_n(t_i), da_n/dt(t_i))\), \( i = 1, 2, .., 4000 \). A linear forward differencing algorithm is used to compute the time derivative of the POD coefficients:

\[
\frac{\partial a_n(t_o)}{\partial t}(t_o) = \frac{a_n(t_o + \Delta t) - a_n(t_o)}{\Delta t}
\]

(4.35)

The advantage of using DT-PIV is that this term is measured directly. Previous work (see Ausseur et al. [10]) has shown that estimating this term using stochastic models is not quite as efficient unless the estimate is of very good quality. Improving state estimates calculated using a simultaneously acquired time-resolved signal of another nature (e.g pressure measurement) can be done using multi-time stochastic estimation techniques as first performed by Ewing and Citriniti [37] and also implemented by Ausseur and Glauser [9] and Durgesh and Naughton [36] which have shown to be much more effective in rebuilding the dynamics in turbulent flows. The DT-PIV technique is suitable in all types of flows from low- to very high-speed flows and there is no need for time-resolved measurements using this identification method.

To determine each one of the constant, linear, quadratic and cubic coefficients one needs to derive as many equations as there are unknowns. In this aim, several moments of Equation 4.34 are taken and ensemble averages are performed using the experimental database; ensemble averaging is denoted by \( \langle . \rangle \), and Einsteinian summation is used for compactness:
\[
\begin{align*}
\langle a_m \frac{da_n}{dt} \rangle &= D_n \langle a_m \rangle + L_{nj} \langle a_m a_j \rangle + Q_{njk} \langle a_m a_j a_k \rangle + C_{njkl} \langle a_m a_j a_k a_l \rangle \\
\langle a_o a_p \frac{da_n}{dt} \rangle &= D_n \langle a_o a_p \rangle + L_{nj} \langle a_o a_p a_j \rangle + Q_{njk} \langle a_o a_p a_j a_k \rangle + C_{njkl} \langle a_o a_p a_j a_k a_l \rangle \\
\langle a_q a_r a_s \frac{da_n}{dt} \rangle &= D_n \langle a_q a_r a_s \rangle + L_{nj} \langle a_q a_r a_s a_j \rangle + Q_{njk} \langle a_q a_r a_s a_j a_k \rangle + C_{njkl} \langle a_q a_r a_s a_j a_k a_l \rangle
\end{align*}
\]

As pointed out in [87], solving this set of equations is equivalent to solving the least squares minimization problem \( X^2 = |B - AX|^2 \) by taking the matrix derivative of \( X^2 \) with respect to \( X \), the matrix of the unknown coefficients. The problem to be solved is finally:

\[
A^T X - A^T B = 0 \quad (4.37)
\]

with

\[
\begin{align*}
B &= [\dot{a}_n(t_1) \; \dot{a}_n(t_2) \; \cdots \; \dot{a}_n(t_N)]^T \\
A &= \begin{bmatrix}
1 & a_j(t_1) & \cdots & a_j(t_1) a_k(t_1) & \cdots & a_j(t_1) a_k(t_1) a_l(t_1) \\
1 & a_j(t_2) & \cdots & a_j(t_2) a_k(t_2) & \cdots & a_j(t_2) a_k(t_2) a_l(t_2) \\
& \vdots & \ddots & \vdots & & \vdots \\
1 & a_j(t_N) & \cdots & a_j(t_N) a_k(t_N) & \cdots & a_j(t_N) a_k(t_N) a_l(t_N)
\end{bmatrix}
\end{align*}
\]

\[
X = [D_n \; L_{nj} \; \cdots \; Q_{njk} \; \cdots \; C_{njkl}]^T
\]

It can be noticed that this method contains redundancy in the resolution of the matrix problem. In matrix \( A^T A \), the terms \( \langle a_j a_k \rangle \) and \( \langle a_k a_j \rangle \) are equal and this is also true for the cubic terms which are repeated. Correspondingly, in matrix \( A^T B \), the terms \( \langle a_o a_p \frac{da_n}{dt} \rangle \) and \( \langle a_p a_o \frac{da_n}{dt} \rangle \) are equal. Hence, several equations are repeated and these terms need to be left out to avoid such redundancy. This amounts to summing \( \sum_{j=1}^{N_{LODS}} \sum_{k=j}^{N_{LODS}} \) instead of \( \sum_{j=1}^{N_{LODS}} \sum_{k=1}^{N_{LODS}} \) and the same applies for the cubic terms.

The size of the problem can become very quickly unpractical especially when
including cubic terms. The number of unknowns is given by the size of matrix X and contains:

- 1 constant unknown $D_n$,

- $N_{LODS}$ linear unknowns $L_{nj}$,

- $\frac{N_{LODS}(N_{LODS}+1)}{2}$ quadratic unknowns $Q_{njk}$,

- $\frac{N_{LODS}(N_{LODS}+1)(N_{LODS}+2)}{6}$ cubic unknowns $C_{njkl}$,

which totals to $1 + N_{LODS} + \frac{N_{LODS}(N_{LODS}+1)}{2} + \frac{N_{LODS}(N_{LODS}+1)(N_{LODS}+2)}{6}$ unknowns.

The following table indicates the size of the matrix problem (i.e. the number of equations to be solved or the number of unknowns) for different number of modes used $N_{LODS}$ in the cases of a quadratic or cubic system. It is seen that the size of the system grows very rapidly especially in the cubic case.

<table>
<thead>
<tr>
<th>Order</th>
<th>$N_{LODS}$</th>
<th>Number of equations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>231</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>861</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>1891</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>5151</td>
</tr>
<tr>
<td>quadratic</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>165</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>455</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1771</td>
</tr>
<tr>
<td>cubic</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>165</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>455</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1771</td>
</tr>
</tbody>
</table>

Table 4.2: Number of equations to be solved for depending on the order of the system and the number of modes retained.
The most limiting factor is however not the RAM memory required to store the matrix and solve the problem but the fact that when $N_{LODS}$ is too large, then $A^TA$ becomes ill-conditioned, meaning that the ratio of the largest to the smallest value of the matrix is too large (greater than $10^{12}$). This is mainly due to the presence of the higher order terms (up to $6^{th}$ order in $a(t)$ in the cubic case) which results in extremely small moment values. Inverting an ill-conditioned matrix can lead to large errors, greatly compromising the quality of the dynamical system. The number of modes kept in the low-order system is therefore always chosen to ensure the problem is well conditioned. This issue restricted the number of modes retained in the modeling to $N_{LODS}=65$ in the quadratic case and $N_{LODS}=12$ in the full modeling with the cubic terms.

This system identification technique can be quite sensitive to many different parameters (number of samples in the ensemble average, truncation level, quadratic or cubic modeling...) but when they are set appropriately, the dynamical system is found to be quite robust as will be seen in the next section. The most time-consuming part of the process (besides acquiring the experimental data) is solving the system identification problem, but in a practical situation, these coefficients are determined off-line. It is therefore only the integration process for getting predicted values that is performed online. This is an extremely fast operation that can be performed in real-time even in high-speed flow control applications. This integration step is described in the following paragraph.

### 4.6.3 Time prediction: the LODS

One of the problems inherent to low-order dynamical systems in fluid dynamics is that a truncation is performed, thereby greatly reducing the dimensions of the problem but also rejecting most of the small scales responsible for dissipating the energy through viscous forces. A common resulting behavior is for the dynami-
cal system to rapidly diverge, lacking dissipative effects. To model the truncated scales and drain the energy in the system, an *ad hoc* artificial viscosity $\nu_a$ is introduced [7] [32] [17] [81] to consistently adjust the linear coefficient throughout all the modes; this has the effect of stabilizing the system and preventing its tendency to quickly diverge. Another non-desired behavior can result from introducing too much dissipation in the model; in that case the system converges to a constant value (as can be seen in Figure 4.39 where the viscosity is purposely set to a large value).

An initial condition on $a_n(t_o)$ for all modes is prescribed: a random instantaneous snapshot measurement from the ensembles is used here but in a practical situation any real-time measurement can be used and a new initial condition can be forced when a measurement is available since the prediction is only effective for short times. From this initial condition, the state of the flow at the following time step is calculated by integrating the following equation:

$$\frac{d a_n}{d t} = D_n + \sum_{j=1}^{N_{LODS}} (1 + \nu_a) L_{nj} a_j + \sum_{j=1}^{N_{LODS}} \sum_{k=j}^{N_{LODS}} Q_{njk} a_j a_k$$

$$+ \sum_{j=1}^{N_{LODS}} \sum_{k=j}^{N_{LODS}} \sum_{l=k}^{N_{LODS}} C_{njkl} a_j a_k a_l$$ (4.39)

The integration scheme used is a 4th order Runge-Kutta method that is described in Appendix B, and which provides a value of $a_n$ for all modes at the next time step. The time-step is tied to the Runge-Kutta integration procedure, and can be set by the user. The stability of the system is also sensitive to this time step, therefore this is a parameter to be tuned when developing the dynamical system to ensure at least short-time stability of the prediction.

One of the disadvantages of using DT-PIV as a means to measure the time derivative of the POD coefficient (as opposed to using a time-resolved measurement - which is not feasible in a high-speed flow - or simulation data) is that the
measured samples of the flow state are all independent due to the limited sampling frequency of the PIV. Therefore, given a initial condition that is part of the ensemble, it is impossible to compare the time series of the predicted POD coefficients, which are time-resolved, to the actual data which is not. There are consequently only a limited number of ways to check that the constructed dynamical system makes physical sense and describes accurately the dynamics of the real system and all are explored and shown following the time series of the predicted POD coefficients:

- The frequency content of the time-series (although short) can be visualized using a wavelet transform. Background on wavelet analysis can be found in Farge’s Annual Review article [38] and also in Mallat’s wavelet tour of signal processing [71]. The author of this manuscript is grateful to Dr. Lewalle at Syracuse University for providing guidance towards implementing continuous wavelet analysis on this data set and providing a program to apply the wavelet transform using both a Morlet wavelet and a Mexican hat wavelet. As could be expected, both transforms showed similar local frequency intensities and only the Morlet wavelet coefficients are shown. A thorough overview on the applications of wavelet analysis (both discrete and continuous) in experimental fluid mechanics by Lewalle et al. [65], details the normalization coefficients used in the present work. The frequency content retrieved from these transforms is then compared to the spectra from the laser doppler anemometry (LDA) measurements performed in the same facility on the Mach 0.6 jet by Hall [51].

- Phase plots of, for example, $a_1$ as a function of $a_2$ or $a_3$ as a function of $a_4$ can show how well the predicted dynamics cover the phase domain of the original data used to train the system. The choice of which mode to plot as a function of which other is guided by the spatial structure of the corresponding eigenfunctions discussed in Section 4.5.2.
A total of 6 low-order dynamical systems are developed at the downstream position of \( \frac{x}{D} = 8 \) to study the influence of the different modeling parameters on the prediction of the dynamics and hence to find which are critical to building a successful dynamical system in a highly turbulent flow. The following parameters have been found to have a strong influence on the results:

- **Cubic or quadratic modeling**: This has a direct consequence on the size of the system, its ability to retrieve the right frequency content, the good- or ill-condition of the matrix to be inverted, and indirectly the amount of TKE retained in the modeling.

- **Truncation level** \( N_{LODS} \): The number of modes kept in the system identification dictates how much of the original TKE is represented. It is also tied to how much TKE is filtered out along with the dissipation scales. The truncation of these scales requires the user to fix an artificial viscosity. The more modes are kept, the least scales are truncated, and the smaller the adjustment needs to be.

- **The number of “training” samples** \( N_s \): The more training samples used, the better conditioned is matrix \( A^T A \), hence the more modes can be used in the modeling. It is usually better to have a number of training samples which is much greater than the number of unknowns in the problem [87]. This constraint was verified in most successful cases.

- **The initial condition** \( t_0 \): The dynamical system can be quite sensitive to the initial condition, especially for the long-time behavior of the system. For short-time prediction however, a given set of identified coefficients can be given a wide range of initial conditions, which is crucial once the LODS is implemented in a flow control scheme where the estimate will need to be periodically corrected.
with new initial conditions as measurements of the true state become available to improve the prediction.

In a first step, the full cubic model (Equation 4.39) was used as the system to be trained with experimental data. The problem being considerably large and the moments matrix becoming very quickly ill-conditioned, only 12 modes are used, which represents 30% of the TKE. To be able to include more higher modes in the ODE training, hence truncating less kinetic energy, a quadratic model was used keeping up to 65 POD modes (64% of the TKE). The following table describes the parameters used in each system:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Resulting System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order</td>
<td>$N_{LODS}$</td>
</tr>
<tr>
<td>LODS 1</td>
<td>cubic</td>
</tr>
<tr>
<td>LODS 2</td>
<td>cubic</td>
</tr>
<tr>
<td>LODS 3</td>
<td>quadratic</td>
</tr>
<tr>
<td>LODS 4</td>
<td>quadratic</td>
</tr>
<tr>
<td>LODS 5</td>
<td>quadratic</td>
</tr>
<tr>
<td>LODS 6</td>
<td>quadratic</td>
</tr>
</tbody>
</table>

Table 4.3: Parameters used in the 6 different low-order dynamical systems.

Cubic models

The two first LODS were developed from the full cubic ODE (Equation 4.39) as from a modeling standpoint, is contains the higher order terms necessary to model the interactions between the mean flow and the fluctuating velocities, since the Reynolds decomposition is used. As discussed previously, these models are limited in the number of modes that can be retained since the moments matrix
becomes very quickly ill-conditioned. Using a maximum of 12 modes out of 4000 (30% of the TKE), 2 dynamical systems were developed with the only difference being the initial condition $a_n(t_o)$, $n=1,\ldots,12$. To prevent the systems from diverging due to the brutal truncation, an artificial viscosity represented by the term $\alpha_a=-0.6$ in Equation 4.39 is needed. All values of $\alpha$ between -0.3 and 0 are found to render the system unstable as well as values smaller than -0.55. The time-series of the 12 first predicted POD coefficients are shown in Figures 4.31 and 4.32 for both dynamical systems. The only difference between the two systems is the initial condition which is given by the POD coefficients from two statistically independent PIV snapshots (number 23 and 35 respectively). It is clear that both
systems are attracted towards the same dynamics even though the initial conditions are different. The predicted dynamics are clearly low-dimensional, as could be expected from the severe truncation of the higher modes, and tend to be drawn towards a sinusoidal limit cycle. To study the velocity field dynamics retrieved by combining the modes, the time series of the predicted low-dimensional velocity field is rebuilt from these predicted POD coefficients using Equation 4.21. From these, the centerline axial velocity time series is plotted and a wavelet transform of this short-time signal is performed to study its instantaneous frequency content, a Fourier transform of such a short time series being inappropriate. Figure 4.33 shows the time series of both LODS’s along with their Morlet wavelet transform.
The dominant frequency appears to be around 500 Hz which is low compared to the measurements performed with a laser doppler anemometer (LDA) by André Hall [51]. A plot of the spectra measured at different downstream positions along the centerline is shown in Figure 4.34. Measurements were performed at a Mach number of 0.6 only up to x/D=6, hence by extrapolation it would seem natural to find dominant frequency content in the vicinity of 1000 Hz at x/D=8, the location under investigation here. It can be concluded that severely truncated cubic models are not able to retrieve the higher frequency content in high-speed flows where the turbulence spectrum is broad and higher modes seem to play an important role.

**Quadratic models**

The next four LODS are developed using the quadratic form of the ODE. The problem being much smaller compared to the cubic model, it is now possible to include many more modes in the system identification procedure. This is done with the aim of including additional smaller scales that are truncated in the cubic model and that seem to be crucial in highly turbulent flows to capture the higher frequency dynamics which are dominant in this flow. The different parameters pertaining to each system are found in table 4.3 presented in the previous section. The modeling equation for the quadratic systems is therefore of the form:

\[
\frac{da_n}{dt} = D_n + \sum_{j=1}^{N_{LODS}} (1 + \nu_a)L_{nj}a_j + \sum_{j=1}^{N_{LODS}} \sum_{k=j}^{N_{LODS}} Q_{njk}a_ja_k
\] (4.40)

LODS 3 is developed using 30 modes and 749 training samples. Figure 4.35 shows the time series of the first 12 POD modes \(a_n(t)\), n=1 to 12. By fixing the added viscosity to \(\alpha_a=0.4\), the quadratic model is able to be contained for a quite long period of time (25 ms) in a bounded domain. Although exhibiting significant periodicity in its dynamics, the frequency content is shown to be much more representative of the actual dynamics existing in such a flow. The wavelet trans-
Figure 4.33: (top) Time series of the predicted centerline fluctuating axial velocity and (bottom) norm of the Morlet wavelet transform of the time series, $x/D=8$.
Figure 4.34: Spectra of the centerline axial velocity component, $M=0.6$, $x/D=1$ to 6, from Tinney (2005).

form (Figure 4.36) shows a dominant frequency at around 1000 Hz, intermittently perturbed by higher frequency and amplitude fluctuations. The presence of high frequency dynamics are a large improvement on the previous cubic systems, which is largely due to the fact that a greater number of modes are kept in the modeling.

LODS 4 shown in Figure 4.37 is built with 48 POD modes and a greater number of training samples but is not able to stay bounded for as long as LODS 3. However it exhibits much higher frequency dynamics than LODS 1 and LODS 2 and the same range of frequencies as LODS 3, around 1 kHz. This can be better visualized in Figure 4.38. The fluctuations of the centerline axial velocity in this figure have a much higher amplitude than that retrieved by the cubic systems which seems to come from the fact that less TKE is truncated from the system in the identification of the ODE coefficients.

Purposely, LODS 5 is prescribed a high artificial viscosity of $\alpha_a=0.6$ to study the consistency of the effect of this \textit{ad hoc} parameter. Indeed it is seen in Fig-
Figure 4.35: Time series of the 12 first predicted POD modes of LODS 3, $N_{LODS}=30$, $x/D=8$.

Figure 4.39 that the POD coefficients quickly converge to constant values due to an excessive action of the artificial viscosity in the system. The process of finding the right viscosity to compensate for the truncation is quite tedious but it is indispensable to prevent the system from either diverging (too low viscosity) or converging towards zero (too high viscosity). However once the parameters of the dynamical system are fixed and the coefficients of the system are identified, the successfully tuned viscosity will be efficient for all initial conditions and at least short time prediction is ensured.

LODS 6 is developed with 65 modes, which represents the maximum size of the
Figure 4.36: LODS 3, (top) Time series of the predicted centerline fluctuating axial velocity and (bottom) norm of the Morlet wavelet transform of the time series, $N_{LODS} = 30$, $x/D = 8$.

quadratic system without it being ill-conditioned. The higher number of training samples enables a better convergence of the statistics and hence a better conditioned matrix problem. This LODS presents the highest frequencies, which is consistent with the finding that the more modes are included in the system, the more realistic the frequency content retrieved in the system. Additionally, very important intermittency is observed in this case with high amplitude fluctuations followed by periods of rather low fluctuations. Figures 4.40 and 4.41 respectively show the evolution of the first 12 POD coefficients and the centerline fluctuating axial velocity along with its wavelet transform. Since the frequency content of the time series in this case is quite intermittent, the wavelet transform does not clearly reveal a dominant frequency. However further analysis of the reoccurrence of a chosen frequency band (by applying a wavelet transform of the coefficients of a chosen frequency) could shed light on the dynamical content of these higher order systems. It is also seen here that the amplitude of the centerline fluctuating
velocity is closer to that found in the original flow (20 m/s) - see Figure 4.17.

**Phase plots**

The study of the frequency content of the predicted velocity field demonstrates that a severe truncation of the higher modes in the case of the cubic modeling prevents any high frequency dynamics from being retrieved in the LODS. For a more direct comparison with the original PIV data and as mentioned in Section 4.6.3, the evolution of the phase between the different POD coefficients can reveal how well the
Figure 4.38: LODS 4, (top) Time series of the predicted centerline fluctuating axial velocity and (bottom) norm of the Morlet wavelet transform of the time series, $N_{LODS}=48$, $x/D=8$.

reconstruction covers the original phase domain. In cases where the dynamics are strongly periodic (e.g. a mixing layer with Kelvin-Helmholtz vortices) then it is found that the POD coefficients are grouped by pairs and the two coefficients are out of phase by $\pi/2$ with respect to one another. The phase representation of such a pair would be a circle centered around the origin and can therefore measure how well the circle is reconstructed by the predicted POD coefficients. In the present case where the dynamics are broadband and no significant periodicity is found in the original data, the phase representation of the original POD coefficients is a cloud of data points uniformly distributed around the origin. It is therefore more difficult to identify a particular attractor that could be rebuilt with the LODS but it is possible to see how well the original domain is covered by the coefficients of a short-time prediction. This can provide a good sense of the quality and more or less realistic nature of the prediction.
Figure 4.39: Time series of the 12 first predicted POD modes of LODS 5, \( N_{LODS} = 65, x/D = 8 \).

Figure 4.42 shows for the two cubic LODS’s the phase relationship between the first two modes. The original data (red crosses) appears to cover a much larger area than that rebuilt by the cubic systems. The dynamics recovered by LODS’s 3 and 4 presented in Figure 4.43 show a much better representation in the phase domain which once again shows the advantage in highly turbulent flows of retaining more modes in the model rather than adding the cubic terms. In Figure 4.44 is plotted the evolution in the phase space of different pairs of coefficients for LODS 6. As in the frequency content investigation, LODS 6 seems to be the system that exhibits the most similarities with the original data.
Dynamical reconstruction

A low-dimensional time-resolved reconstruction of the 3-component velocity field can be obtained by projecting the LODS time dependent POD coefficients onto the original eigenfunctions as given by:

\[ u_i(\vec{x}, t) = \sum_{n=1}^{N_{LODS}} a_n(t) \phi_i^{(n)}(\vec{x}) \]  

(4.41)
Animations of the axial and in-plane components of the fluctuating velocity of each of the 6 dynamical systems were computed to illustrate the features of the fluctuating velocity field in real-time as the dynamics are predicted from the prescribed initial condition. These animation files have been compressed and can be viewed or downloaded online at the following URL:

http://www.ecs.syr.edu/faculty/glauser/GradStudents/JeremyPinier

### 4.6.4 Recommendations for high Reynolds number LODS development

- The moments method for systems identification has shown that it can be applied to low as well as high Reynolds number flows. The only shortcoming of low order modeling at high Reynolds numbers, and this is not specific to this
method, is that the smaller scales that are truncated in the modeling play a considerably more important role for the dynamics of the flow as in the case of low Reynolds number flows. Indeed they are responsible for dissipating the TKE produced at the larger scales. Since production of TKE is much larger in highly turbulent flows, the dissipative scales become much more im-
Figure 4.44: LODS 6, $N_{LODS}=65$, phase representation of (a) $a_2 = f(a_1)$, (b) $a_{10} = f(a_9)$ and (c) $a_{26} = f(a_{25})$, $x/D=8$. 
important and truncating them presents challenging consequences. This is not as much an issue in lower Reynolds number flows where for one part a lower number of modes captures a larger part of the TKE and also where the dissipative scales are not as crucial since production is not as important. Hence, in the low Reynolds number flows, a cubic model could be more effective at predicting the dynamics than a quadratic one. It is however shown here that modeling the interaction of the turbulence with the mean flow (through the cubic terms) is not as crucial as using as many modes as possible (hence as much TKE as possible) in the development of the dynamical systems. It is therefore recommended that quadratic models with a larger number of modes be used when developing LODS’s in high-speed and highly turbulent flows.

- The choice in the truncation level has shown to be a crucial parameter and a careful study of the spatial distribution of the POD modes is highly recommended to prevent the system from being unstable. It was shown that the spatial POD modes can be associated to others by similarity of spatial features. This finding can guide the choice in the level of truncation to prevent “groups” of modes from being parted. This leads to the following recommendation, which is specific to axisymmetric flows.

- When the measurement permits, it is suggested not to assume axisymmetry a priori and to learn from applying the 2D-POD to all three components of velocity instead of applying the Fourier transform in the azimuthal direction followed by the POD in the radial direction. In that case the phase relationship between the components is lost and the interpretation of the spatial POD modes becomes incomplete and perhaps misleading. Indeed interpreting the spatial structure of the POD modes has shown to be important in
the choice of truncation level as well as insightful for the physical interpretation of the dominant structures in the flow in terms of turbulent kinetic energy. PIV measurements as well as CFD simulations enable estimations of the entire field, it can therefore be enriching to study the 2D-POD results without assuming axisymmetry in at least a first step.

4.6.5 Next step: Application to closed-loop flow control

Given initial conditions, the prediction of the future state of the flow using the evolution ODE’s with the coefficients identified off-line along with the Runge-Kutta integration is a computational operation that can be performed faster than the evolution of the flow. The accuracy of the prediction has shown to be reasonably rich in high frequency information considering the large truncation applied to the system. It is therefore a valid candidate to be used as a short-time predictive model in a closed-loop flow control architecture. The use of models in closed-loop control is dual:

(1) It can be used as a plant to develop a controller off-line by replacing the “real system” (i.e. the flow) by the derived LODS and simulating control laws that stabilize the system or bring it to its “desired state”. The developed controller can then be tested on the real system and the degree of success of the control laws depends directly on the quality of the empirical model used during the simulation.

(2) The model can also be used on-line as a predictive estimate of the state of the flow. It is then associated with a measurement performed on a larger time scale that can correct the prediction or prescribe it a set of new initial conditions periodically, since the predictive capability is limited to short durations before the system diverges. This type of architecture is commonly found in controls where both a measurement and a prediction are typically used in a Kalman filter to estimate the state of the flow at the following time step.
In both cases however, the control input needs to be explicitly included in the model to account for the changes due to the control input. This task is not straightforward in fluid mechanics problems (as compared to solid mechanics problems) since the actuation has often very subtle effects on the flow and the response from the system is strongly non-linear. Future implementation of the model in a closed-loop control architecture would have to address this issue. Several ideas have been implemented to explicitly include the actuation term in the model ODE. Carlson [25] has derived a velocity decomposition that includes the effect of the actuator input, as follows:

\[
\begin{align*}
    u_i(\vec{x}, t) &= U_i(\vec{x}) + \zeta(t)\bar{u}_i(\vec{x}) + u_i'(\vec{x}, t) \\
    \text{(4.42)}
\end{align*}
\]

where \( \bar{u}_i(\vec{x}) \) is the spatial effect of the actuation and \( \zeta(t) \) the time dependent amplitude of the forcing. The experimental procedure that enables this decomposition is described in detail in [25]. This decomposition is then analytically substituted in the Galerkin projection to derive a new ODE that includes explicit actuation terms. The full non-linear derivation results in an ODE of the following form (with summation on double indices):

\[
\begin{align*}
    \frac{da_i}{dt} &= A_{ij}a_j + B_{ijk}a_ja_k + C_{ij}a_j\zeta + D_i\zeta + E_i\zeta^2 + F_i\frac{d\zeta}{dt} \\
    \text{(4.43)}
\end{align*}
\]

where \( A_{ij} \) and \( B_{ijk} \) are the linear and quadratic coefficients as would be found from a Galerkin projection of the Navier-Stokes equations and \( C_{ij}, D_i, E_i, F_i \) are the extra coefficients of the terms that contain the actuation input. The moments method as described in this chapter can now be applied using data with and without actuation to identify these coefficients. The state of the flow can then be predicted in a closed-loop control case where the amplitude of the actuation \( \zeta(t) \) is known at all instants. A similar solution to the problem of including explicit actuation terms in the ODE has been implemented by Caraballo et al. [24] with promising results in the case of the compressible cavity flow. In a different manner,
but with the same concern of accounting for the actuation input, Camphouse [23] has recently derived a “Split POD” method that builds two orthogonal sets of POD eigenfunctions, one for the non-actuated flow and the other orthogonal to the first, isolating the effects of the forcing only. The reader is referred to [23] for the details of the implementation of this technique.
Chapter 5

Design of active flow control devices: towards jet noise reduction

The findings from the experiment relating the near-field to the far-field pressure (Chapter 3), the development of the low order dynamical model of the flow and the lessons learnt from previous applications of closed-loop control of turbulent flows (see Pinier et al. [88]) are used as guidelines for the design of an actuating device that would have the potential of controlling the far-field sound produced by the high-speed jet in a closed-loop architecture. The ideas towards jet noise reduction that have emerged from this work are laid out, the design constraints are addressed and the final system is presented. The implementation of the closed-loop control described in the previous chapter using this device is left for a future investigation focusing on integrating the LODS for active flow control on the high-speed jet.

5.1 Motivation

It has been shown through direct numerical simulation (DNS) along with adjoint flow control methods [109] that free shear flows can be controlled to be made quiet. The difficulty arises from the fact that the difference in the spatial dynamics of the flow structures between a loud and a quiet flow is almost unnoticeable [109]. The aeroacoustic process that creates sound is undeniably subtle therefore a sim-
ilarly subtle flow control strategy needs to be sought. The jet flow needs to be controlled without being denatured, the main feature of which (i.e. thrust) would be impeded. Many passive or active flow control studies have been carried out on the axisymmetric jet flow. Ranging from tabs [19] (or chevrons) to lobe mixers [72] and from fluidic impinging jets [2] to plasma actuators [94], the techniques are numerous and the effects on the jet flow characteristics are very dependent on the type of actuating device used. In reality, the constraints to designing flow control actuators are much more restrictive since even a small impact on thrust will prevent the technology from reaching the application stage. Indeed, the jet engine industry has been quite conservative in their efforts towards jet noise reduction and only chevrons have reached the commercialization stage. The focus therefore needs to be directed towards finding “non destructive” ways to slightly manipulate the flow and use flow based amplification to optimize the impact of the actuation on the flow dynamics. In the case of aeroacoustic flow control, it is envisioned that a modification of the near-field pressure around the jet could inhibit noise propagation without having to modify the dynamics of the flow itself. Aside from acoustic control of the jet flow [76], very few have studied the effects of a slight disturbance input to the axisymmetric jet flow near the lip. The present jet flow control device has therefore been designed with this fundamental constraint in mind. A circular array of synthetic jet actuators (zero net-mass-flux) with exit velocities on the order of 50 - 80 m/s was designed and integrated to the jet nozzle.
5.2 Concept: Jet noise reduction by manipulation of the near pressure field.

5.2.1 Design constraints for closed-loop control devices

Several fundamental constraints need to be at least addressed and at best optimized when designing flow control devices. These constraints should all be intended to efficiently fulfill one criterium: the control objective.

- **Define the control objective**: Minimize the sound perceived in the far-field region of the high-speed jet. An understanding of the physics underlying the process of noise generation in the jet is fundamental for defining and finding devices that have the necessary degrees-of-liberty to perform the task of modifying the structures responsible for noise production. It can also guide the most efficient location for the actuators to have the largest influence on the control objective. However a full understanding of the dynamical mechanisms inside the jet flow responsible for sound generation, although interesting from a fundamental point of view, is not required in a flow control application. Furthermore, as is reported in Chapter 3, a better appreciation of the mechanisms in the pressure field surrounding the jet flow may be enough to find ways to inhibit the sound propagation mechanism. It has been indeed shown from the understanding gained in the first experiment that an active “mode 1” forcing of the pressure field to counteract the high amplitude axisymmetric events (mode 0) could lead to inhibiting the sound propagation process. The actuators have therefore been designed to be able to explore different modal actuation configurations (in particular mode 1).

- **Observability**: Since only a very small fraction of the turbulent kinetic energy in the flow is dissipated into acoustic energy or sound pressure waves, noise generation in aeroacoustics is said to be a very “inefficient” process,
which explains the difficulty in finding the sources of sound in a flow even with the most reliable measurement techniques. From given far-field acoustic measurements, it is therefore almost impossible to determine the inner state of the flow at the time the sound was created, which makes the system minimally observable and creates a greater challenge for closed-loop control. This lack of observability needs to be compensated for, either by multiplying the sensing locations (which are in practice limited in the case of a free jet) or by improving the controllability of the system.

• **Controllability**: This criterium measures the ability, given an actuating system, to force the state of the flow into various states of its configuration space. To maximize the impact on the flow or the surrounding pressure field, actuators need to be located at the lip of the jet so that even a minimal input to the flow is amplified through shear layer instabilities as they develop downstream. In high-speed flows, the frequency response of the actuators needs to be at least of the same order of magnitude as the dynamics of the flow if one wishes to actively control the flow on the same time scales as the important flow features. The frequency range of the synthetic jet actuators (100 Hz - 2500 Hz) covers the entire range of dominant frequencies measured in the near pressure field. These actuators have also shown to produce jets on the order of 50 m/s which could be enough to actively trigger the shear layer at the nozzle exit and modify the near pressure field without changing the nature of the jet flow.

• **Cost efficiency**: A study of the efficiency of the control needs to be performed to make sure that the amount of energy introduced in the system is not higher than the amount of energy saved through the flow control.

• **Robustness, practicality**: The conditions in which flow control systems actually operate are very often highly unsteady and could be quite different than the conditions under which the system has been designed and tested.
Robustness of the control to changes in operating conditions is therefore crucial. Additionally, the main issues that could be encountered on a real application need to be addressed for the control system to be practical. The synthetic jet actuators have shown that they can withstand very rough in-flight operating conditions [95] and are therefore great candidates for flow control on high-speed jets. Additionally, the piezoelectric diaphragms operate at resonance so the power output to power input ratio is optimal.

5.3 Synthetic jet actuator development

5.3.1 Helmholtz resonance

Synthetic jet based actuators rely on the Helmholtz resonance of a slotted cavity to create a free jet at the output of the slot. This continuous jet can then be actively amplitude-modulated and used in a closed-loop control scheme to perform high frequency actuation. Each synthetic jet system consists of two subsystems that directly affect the frequency response of the system: a slotted cavity and an oscillating piezo-electric diaphragm. Both of these subsystems have their own resonance frequencies and when assembled exhibit different overall resonance frequencies. The use of resonating devices makes the technique very cost efficient since a minimal input to the system forced at its resonance frequency has a maximum output. Helmholtz resonators are also considered compact resonators [53].

Given a cavity with a slot area $A$, neck length $L$ and volume $V$, the resonance frequency $\Omega_o$ can be shown to be proportional to:

$$\Omega_o \propto \sqrt{\frac{c_o^2 A}{L V}}$$

(5.1)

where $c_o$ is the speed of sound. However the accurate resonance frequency of the cavity is found to be very much dependent on its geometry. To maximize the output of the synthetic jet, many different parameters can be modified to greatly affect
the output jet velocity. An empirical investigation using hot-wire measurements at the output of the slot has been carried out varying the cavity volume and neck length [3]. Figure 5.1 displays the final design of the cavity that enables maximum output velocities on the order of 80 m/s (40% of the exit velocity), which is believed to be enough to slightly disturb the developing shear layer.

5.4 Actuator bank design

Figures 5.1 and 5.2 present computer aided design (CAD) images of the actuator glove mounted on the jet nozzle. Each individual slot is the output of a Helmholtz resonating cavity forced by two piezoelectric actuators acting in phase at their natural resonance frequency as described in the previous section. The cavity size is designed to maximize the output flow from the slot. Although this type of actuator has a zero net-mass flux of air through the slot, it is able to create a synthetic jet at the output. The mechanism was described thoroughly by Glezer.
and Amitay [47]. The oscillations can be separated into a blowing phase and a suction phase. During the blowing phase, the flow is pushed straight out of the slot with high momentum and during the suction phase, the flow comes in from the sides where the momentum is the lowest. The frequency of the oscillation is matched with the resonance frequency of the actuating disc which is here 2700 Hz, but the only purpose of this frequency is to allow the synthetic jet flow to form at the exit of the slot. The frequency itself is not significantly sensed in the output flow. This high frequency can drive the actuators in an open-loop manner or it can be amplitude-modulated by a frequency more tailored to the particular flow under study such as the Strouhal of the column mode instability of the jet. In a closed-loop scheme, the input signal to the actuators would modulate the high frequency carrier that is needed to drive the actuators.

Eight individual slots are located at 0.6 mm from the lip of the jet (as seen in Figure 5.2) and cover the entire circumference of the nozzle exit to provide ac-
tuation uniformly around the out-coming shear layer. Each synthetic jet can be independently controlled to provide actuation patterns in the form of modes 0, 1, 2 and 4. In a first open-loop step, either steady or flapping modes can be used as has been demonstrated by Samimy et al. [94] using individual plasma actuators. In the next step, closed-loop control will be applied, where the actuation pattern at the lip can be guided by information from sensors located further downstream in the loud region of the jet.

The piezoelectric actuators are thin brass discs (27 mm diameter and 0.22 mm thickness) on which is attached a ceramic disc that strains when a voltage is applied to it. A prototype of the actuating glove was then designed using CAD and built using stereolithography. This technology (as compared to computer numerical control (CNC) machining or such) is very well adapted for creating laboratory prototypes since it is fairly low-cost and the accuracy of the models is very fine. The material used is usually a hard and durable, resin based material (here SO-MOS 11120 Watershed). The only drawback would be when the application is conducted at high temperatures in which case a metal material is more appropriate. For preliminary studies, such a technology is certainly beneficial. Since the diaphragms are high frequency vibrating membranes, a significant amount of sound is produced at their resonating frequency. An acoustic shield has therefore been designed (as seen in Figure 5.3) to prevent most of the actuator sound from propagating to the far-field, where the aerodynamic jet sound meant to be minimized. The frequency of the forced diaphragms being known, it is also possible to easily separate the sound coming from the actuators and that of the flow itself in order to study the ability of these actuators to reduce the far-field perceived sound. The synthetic jets are designed to be slightly inclined towards the jet (at an angle of 45°) to only manipulate the developing shear layer as it is convected downstream and use flow amplification since the shear layer at the exit has been shown to be very receptive to any disturbance [76]. The ultimate aim is, through closed-loop
control, using downstream information in real-time, to perform the subtle changes in the near-field pressure around the jet that would inhibit sound propagation. As described in the preliminary experiment (Chapter 3) and as shown by Hall et al. [52], azimuthal mode 0 (the axisymmetric mode) computed from near-field pressure is the mode that correlates best with the far-field sound and the presence of azimuthal mode 1 inhibits the propagation of sound to the far-field. Therefore it could be beneficial to seed the near pressure field around the nozzle exit with a mode-1 type forcing using downstream information in real time to adjust the frequency of the control input.
Chapter 6

Conclusions

The foundations for a first closed-loop active flow control experiment on the high-speed jet with the aim of reducing far-field noise have been laid out and the results reported in this document form the initial building blocks.

A first experiment carried out in the Syracuse University anechoic chamber and jet facility related the instantaneous near-field pressure in the Mach 0.85 jet to the sound sensed in the far-field (75 jet diameters). The original information learnt from correlating these signals has been quite abundant and the main findings are:

- The location of the loud region in the jet has been identified to be between $x/D=6$ and $x/D=10$, after the collapse of the potential core, consistent with findings in previous aeroacoustic jet research.

- The near pressure field is truly low-dimensional but no dominance of either the axisymmetric mode (Fourier mode 0) or the antisymmetric mode (Fourier mode 1) has been found.

- The cross-correlation between the individual Fourier modes of the near-field pressure and the far-field sound has shown that the axisymmetric mode alone (mode 0) has the largest normalized correlation (above 30% at a polar angle of $\phi = 15^\circ$) with the far-field. This is true for all polar angles and all downstream positions measured in the experiment. Furthermore, the antisymmetric part of the near-field pressure has shown to decrease its overall
correlation with the measured sound, which indicates that an active forcing of the near pressure field in an antisymmetric manner could inhibit the propagation of sound.

From this experiment is pointed out the zone of interest for aeroacoustic flow control, where a low-order dynamical system of the jet flow is built using direct measurements of the acceleration field. In this aim, a dual-time PIV experiment was designed, set up, aligned and calibrated to collect a large database of the velocity field and the Eulerian acceleration field from x/D=3 to x/D=10, simultaneously sampled with the far-field sound. Using low-dimensional methods (POD), the size of the problem is significantly reduced to enable the development of low-order models of the flow. These predictive models show a good agreement with the original data when the truncation of the system is minimized. It is found that in highly turbulent flows like this one ($Re \approx 700,000$) it is beneficial to retain more modes in a quadratic model than to increase the order of the system, which restricts the number of modes that can be used in the modeling. The higher modes are found to be responsible for much of the higher frequency content in the models, which make them more representative of the actual flow.

A preliminary design of a synthetic jet based active control device is achieved using the physical insight gained from the first experiment as to the propagative nature of the near-field pressure. Guidance towards its implementation in a model-based closed-loop flow control architecture is given for future studies.
6.1 Future Work

6.1.1 Develop dynamical systems for all x/D

Only one of the twelve measured downstream positions has been exploited in the present work. This initial study was performed to find the optimal parameters that enable accurate representations of the flow in a low-order model. The lessons learnt now guide the way towards implementing the technique at all downstream positions for a complete picture of the low-order dynamics of the high-speed jet flow.

6.1.2 Wavelet analysis of the LODS’s

An in depth wavelet analysis of the predicted time series could represent an effective diagnostics tool to investigate the frequency content and its degree of similarity with the original data. The time series being short (since only short time prediction is feasible in a highly turbulent flow), wavelet analysis has shown to be an excellent substitute to Fourier analysis that is unable to extract accurate frequency information from short time series. In cases where the dynamics exhibit more intermittency and the wavelet coefficients are not able to extract global frequencies, a second wavelet transform can be applied at a chosen frequency level to examine the reoccurrence of intermittent high frequencies in the jet. It is indeed conceivable that the very short time duration and high intensity events could be at the origin of much of the far-field sound and a correlation of the flow dynamics as captured by the dynamical system with the far-field sound could give such insight.

6.1.3 Further exploit the acceleration and far-field sound database

In the reported work, the POD was applied to the consecutive velocity measurements to compute directly the time derivative of the POD coefficients. The data-
base of the raw acceleration field from 3 to 10 jet diameters downstream simultaneously sampled with the far-field sound is bound to contain rich information about the mechanisms in the flow that create large amounts of sound in the far-field. Cross-correlations between the instantaneous acceleration and the far-field pressure could indicate the types of events in both the velocity and acceleration fields that would be responsible for noise generation. Furthermore cross-correlations of each individual POD mode expansion coefficient with the far-field sound could indicate if one particular structure represented by its spatial eigenfunction is a dominant source of sound in the jet.
Bibliography


Appendix A: Phase angle plots

The following plots show the phase angle as defined in Chapter 3, from which was calculated the velocity of the information traveling from the near- to the far-field region shown in the table. The linear parts of the phase angle plots also show the frequency range where the near and far-field signals are coherent.
Figure 6.1: Plots of the phase angle of the cross-spectra between the near- and far-field pressure, far-field microphones at $\phi = 15^\circ$, $60^\circ$ and $90^\circ$ (left to right), $x/D=1, 2, 3, 4$ and 5 (top to bottom).
Figure 6.2: Plots of the phase angle of the cross-spectra between the near- and far-field pressure, far-field microphones at $\phi = 15^\circ$, 60$^\circ$ and 90$^\circ$ (left to right), $x/D=6, 7, 8, 9$ and 10 (top to bottom).
Appendix B: Fourth order Runge-Kutta integration scheme

The 4\textsuperscript{th} order Runge-Kutta integration scheme (a.k.a. RK4) is an iterative method for approximating the solution of an ordinary differential equation.

Let’s define the function $f$ as being the time derivative of the POD coefficient (i.e. the right-hand-side of Equation 4.39):

$$f(a_n(t)) = D_n + \sum_{j=1}^{N_{\text{trunc}}} (1+\nu_a) L_{nj}a_j + \sum_{j=1}^{N_{\text{trunc}}} \sum_{k=1}^{N_{\text{trunc}}} Q_{njk}a_ja_k + \sum_{j=1}^{N_{\text{trunc}}} \sum_{k=1}^{N_{\text{trunc}}} \sum_{l=1}^{N_{\text{trunc}}} C_{njkil}a_ia_ka_l$$

(6.1)

Given a set of initial conditions $a_n(t_o), n = 1..N_{\text{LODS}}$, the set of predicted values for the next time step is computed by using an average slope, given by:

$$a_n(t_o + h) = a_n(t_o) + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

(6.2)

where $h$ is the time step length and $k_1 = f(a_n(t_o))$, $k_2 = f(a_n(t_o) + \frac{1}{2}hk_1)$, $k_3 = f(a_n(t_o) + \frac{1}{2}hk_2)$ and $k_4 = f(a_n(t_o) + hk_3)$. The process is then reiterated updating the initial condition with the new set of values.