Example

\[ P_A + \frac{1}{2} \rho V_A^2 + \frac{1}{2} \rho g h_A = P_B + \frac{1}{2} \rho V_B^2 + \frac{1}{2} \rho g h_B \]

\[ \frac{\partial}{\partial y} h_B = \frac{1}{2} \frac{\partial}{\partial y} V_B^2 \]

\[ V_B = \sqrt{2g h_B} \]
Step #3: in most (not all) problems, you can solve them by applying

a) Bernoulli equation

b) Conservation of energy
Applications of Bernoulli equation

Some general guidelines you can use when applying Bernoulli equation:

Step #1: Ask yourself whether you can assume
   a) inviscid?
   b) incompressible flow?
   c) steady?

Step #2: If step #1 is OK, then draw a couple of streamlines. Make sure they start/end at points where information are known/asked for.
Euler equation

\[ \frac{\partial p}{\partial t} + g \frac{\partial z}{\partial t} = g \frac{v^2}{R} \]

\[ \frac{\partial}{\partial t} (p + g\rho z) = g \frac{v^2}{R} \]

Let's assume that we can neglect gravity

\[ \frac{\partial p}{\partial t} = g \frac{v^2}{R} \]
This is an excellent example where the Bernoulli number is different for different standards.

\[ P - P_0 = \rho g (H - z) \]

\[ P_1(z) = P_{atm} + \rho g (H - z) \]

\[ \beta_1 = P_1 + \frac{1}{2} \rho g v^2 + \rho g z \]

\[ = \rho_{atm} + \rho g H - \rho g z + \frac{1}{2} \rho g v^2 - \frac{\rho g z}{2} \]

\[ = \rho_{atm} + \rho g H - \frac{1}{2} \rho g v^2 \]
Euler's equation with gravity

$$\frac{\partial}{\partial x} (p + sg \: z) = g \frac{v^2}{R}$$

Let's apply equation at station 0

$$\frac{\partial}{\partial z} (p + sg \: z) = 0$$

$$\frac{\partial p}{\partial z} + sg = 0$$

$$\frac{2p}{\partial z} = -sg$$
Recall example done in class

Assume frictionless
Euler's equation at 2:
\[ \frac{dp}{dh} = 8 \frac{V_2^2}{R_2} \]

Since \( R_2 \to \infty \), \( \frac{dp}{dh} \bigg|_2 = 0 \)

Apply Bernoulli between 1 and 2:
\[ \frac{P}{\text{atm}} + \frac{1}{2} \frac{V^2}{g} = \frac{P_{\text{atm}}}{\text{atm}} + \frac{1}{2} \frac{V_2^2}{g} \]

So, \( V_2 = V \).
Apply Bernoulli equation along streamlines between (3) and (A)

\[ P + \frac{1}{2} \rho U_{\infty}^2 = P + \frac{1}{2} \rho U_A^2 \]

between (3) and (B)

\[ P + \frac{1}{2} \rho U_{\infty}^2 = P + \frac{1}{2} \rho U_B^2 \]

\[ P_A + \frac{1}{2} \rho U_A^2 = P_B + \frac{1}{2} \rho U_B^2 \]

\[ (P_A - P_B) = \frac{1}{2} \rho (U_B^2 - U_A^2) \]

Solving for \( U_{\infty} \)
Lecture 11/2/06

Exam #2 11/17  chapters 4, 5, 6

Pitot-static tube: device used to measure velocity

V∞

A

B

stagnation streamline

Δh

zm