(2) Surface forces: forces acting on the boundaries of a medium through direct contact.
Another way (much longer)

\[ R = \sqrt{\frac{A}{\pi}} \]

\[ dV = (dr)(rd\theta)(dy) \]

\[ dF_B = \mathbf{g} \cdot dV \]

\[ \frac{2}{R_B} = \iint_H \mathbf{g} \cdot \mathbf{r} \, r \, dr \, d\theta \, dy \]

\[ A = \pi R^2 \Rightarrow R = \sqrt{\frac{A}{\pi}} \]
\[ \mathbf{dF}_B = \mathbf{g} \mathbf{dV} \]

\[ \mathbf{dV} = A \, dy \]

\[ A = 1 \, \text{ft}^2 \]

\[ \mathbf{dF}_B = \mathbf{g} \cdot A \rho \, dy \]

\[ \mathbf{F}_B = \int_{0}^{H} \left( A \rho \right) \mathbf{g} \cdot dy \]

\[ H = 10 \, \text{ft} \]

\[ \mathbf{F}_B = A \rho \mathbf{g} \int_{0}^{H} S_0 \left( 1 + ky \right) \, dy \]

\[ = A \rho S_0 \left( H + k \frac{H^2}{2} \right) \]
\[ \delta \vec{F}_b = g \delta \vec{u} \]

Net body force

\[ \vec{F}_b = \iiint \vec{dF}_b = \iiint g \delta \vec{u} \, dv \]

\[ = \iiint g \delta \vec{u} \, dxdydz \]

\[ = \iiint g \delta \vec{u} \, Ardz \, rdv \]
Example: a column of saline solution

\[ A = 1 \text{ ft}^2 \quad \rho = \rho_0 (1 + ky) \quad 0.10 \text{ ft}^{-1} \]

\[ H = 10 \text{ ft} \]

\[ L = 1.94 \text{ slug/ft}^3 \]

Compute net body force acting on column
Consider gravity \( \vec{g} \) (acceleration or force per unit mass) acting on a fluid element of volume \( dV \) and density \( \rho = \rho(x) \).

\[ \text{body force acting on } dV = \rho \vec{g} \frac{dV}{\text{mass}} \]

Check unit:

\[ [g] \frac{[5V]}{[\rho]} \frac{[x]}{[x]} = \frac{\text{kg}}{m^3 \cdot s^2} \cdot \frac{m}{m} = \text{kg} \frac{m}{s^2} = \text{N} \]
Categories of forces in fluid mechanics

There are 2 types of forces acting on a fluid element: body forces and surface forces.

1. Body forces: forces developed without physical contact and distributed over volume of fluid. Examples are gravitational force and electromagnetic force.
Dimension / units

SI (Systeme International)
\[ [M] = \text{kg} \quad [L] = \text{m} \quad [T] = \text{sec} \quad [\theta] = \circ K \]

BG (British Gravitational)
\[ [F] = \text{lb} \quad [L] = \text{ft} \quad [T] = \text{sec} \quad [T] = \circ R \]
Equation of streamline is \( 2cy = 16 \)

\( u = x \)

\( v = -y \)
\[ \int \frac{dy}{y} = - \int \frac{dx}{x} \]

\[ \ln y = - \ln x + C_1 \]

\[ \ln y + \ln x = C_1 \]

\[ e^{\ln xy} = e^{(C_1)} \]

\[ xy = C \]

Determine the value of \( C \) from \( IC \)

\[ (2)(8) = C \implies C = 16 \]
Solve ODE using method of separation of variable

\[ \frac{dy}{dx} = -\frac{y}{x} \]

\[ \frac{1}{y} dy = -\frac{1}{x} dx \]

\[ \int \frac{dy}{y} = -\int \frac{dx}{x} \]

\[ \ln y \bigg|_g^x = -\ln x \bigg|_2^x \]

One way of doing it
Example: construction of streamline given \( \vec{V} = \vec{V}(x,y) \)

Given: \( \vec{V} = (x) \hat{i} + (-y) \hat{j} \)

Find: streamline equation passing through point \((2, 8)\)

Analysis: \( \frac{dy}{dx} = \frac{\vec{v}}{\vec{u}} = -\frac{y}{2x} \)
Define \( \mathbf{V} = u \mathbf{i} + v \mathbf{j} \)

\[ \nabla \times \mathbf{V} = 0 \]

\[ \frac{\partial u}{\partial y} = 0 \]

\[ \frac{\partial v}{\partial x} = 0 \]

\[ \int u \, dy - v \, dx = 0 \]

\[ \frac{dy}{dx} = \frac{v}{u} \]

1st order ODE for \( y = g(x) \) need IC
Equation for streamlines in 2D flows

\[ ds = dx \hat{i} + dy \hat{j} \]

From definition of streamline, we have

\[ \nabla \times \vec{V} \times ds = 0 \]