<table>
<thead>
<tr>
<th>Elevation (miles)</th>
<th>mean free path (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>60</td>
<td>4</td>
</tr>
<tr>
<td>100</td>
<td>2000</td>
</tr>
</tbody>
</table>

Fluid particle: a collection of molecules of size 5V.
Continuum model

This model assumes that the fluid is continuously distributed in the region of interest.

A method used to determine whether the continuum model is valid is to compare the typical dimensions of the problem to the mean free path of the molecules—want at least 1,000 times larger.
Non-dimensionalize

\[ V_x^* = \frac{V_x}{\sqrt{mg/k}} \quad t^* = \frac{t}{\sqrt{mg/k}} \]

Then the solution is

\[ t^* = \frac{1}{2} \ln \left( \frac{1 + V_x^*}{1 - V_x^*} \right) \]

\[ V_x^* \to 1 \quad \text{as} \quad t^* \to \infty \]

\( \text{Terminal speed is} \quad V_x = \sqrt{\frac{mg}{k}} \)
From Table of integral

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left( \frac{a + u}{a - u} \right)$$

Set \( u = V_x \)

\( a = \sqrt{\frac{mg}{k}} \)

After "lots" of algebra

$$\frac{1}{2 \sqrt{\frac{mg}{k}}} \ln \left( \frac{\sqrt{mg/k} + V_x}{\sqrt{mg/k} - V_x} \right) = \frac{k}{m} t$$

Implicit expression for \( V_x \) as a function of time.
ODE for $V_x = V_x(t)$. Need Initial Condition (IC), i.e. $V_x(t=0) = 0$

Use Method of separation of variables to solve ODE

$$m \frac{dV_x}{dt} = mg - kV_x^2$$

$$\left[ \frac{m}{mg - kV_x^2} \right] dV_x = \left( \frac{k}{m} \right) dt$$

$$\Rightarrow \int^{V_x}_0 \frac{dV_x}{mg - kV_x^2} = \frac{k}{m} \int_0^t dt$$
Review of Differential Equation

Example: terminal velocity

e.g. jump out of A/C with a parachute

\[ F_D = kV_x^2 \quad k = \text{constant} \]  

(depends on geometry, must be determined experimentally)

\[ \sum F_x = m \frac{dV_x}{dt} + mg - kV_x^2 = m \frac{dV_x}{dt} \]
Review of Dynamics + ODE

Example

\[
\begin{align*}
\sum F_x &= m \frac{dV_x}{dt} \\
F - \mu mg &= m \frac{dV_x}{dt}
\end{align*}
\]
Lecture 8/31/06

Fluid Mechanics is a branch of classical mechanics. The basic laws are:

1) Conservation of mass
2) Linear/Angular momentum
3) First/Second law of Thermo
4) Additional relations (e.g. \( p = \gamma RT \))

\[ \text{MAE - 341} \]